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Interim Report
No. I-1804-1

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STATISTICAL METHODS
IN
INITIATOR EVALUATION

FC

Prepared for

PICATINNY ARSENAL

Samuel Feltman Ammunition Laboratories

Fuze Development Laboratory

ORDBB-TP3

Department of the Army Contract No. DA-I-36-034-501-ORD(P)-38
Department of the Army Project No. 505-01-003Z
Office, Chief of Ordnance Project No. TA1-2707AU

THE FRANKLIN INSTITUTE
Laboratories for Research and Development

NICOL H. SMITH, Director

Philadelphia 3, Pennsylvania

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Interim Report
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STATISTICAL METHODS IN INITIATOR EVALUATION

by
Carl Hammer

May 1, 1955

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FOREWORD

Since October 1950, The Franklin Institute, Laboratories for Research and Development, has made a study of power sources, initiators and other components for electric fuzes. Originally sponsored by the Office of the Chief of Ordnance, the work since December 1953 has been performed for Picatinny Arsenal, ORDBB-TP1. The program consists of several phases:

(a) A review and evaluation of current work on electric fuzes and their components.

(b) Research and development on electric power sources and fuze components,

(c) Development of instrumentation for the evaluation of electric initiators, and

(d) Evaluation of electric initiators.

Two series of reports are issued on this contract. The information gathered in the Review (identified by the letters MR) receives wide distribution. Progress of experimental work performed at The Franklin Institute Laboratories is contained in the second series of reports identified by the letters MT. In addition to these periodic reports, Interim Reports summarize special studies such as this one.

One of the principal objectives of our phase to evaluate electric initiators is to describe the performance of each initiator for the user's benefit. The statistical methods described in this report were developed as tools to be used in describing an initiator's performance. We hope the report will not only indicate the extent and nature of the problems involved, but adequately describe our present solutions. Copies of the report may be obtained from the Armed Services Technical Information Agency or Picatinny Arsenal, Samuel Feltman Ammunition Laboratories.

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ABSTRACT

The analysis and the evaluation of test data on the functioning of initiators has been carried out at The Franklin Institute Laboratories by means of techniques that are based on statistical principles. We can never pick up an initiator and be able to assert that we know with certainty it will function upon application of a given input, or that its functioning time is less than a given number of microseconds. However, we can, after analyzing sufficient data, say that 999 times out of 1000 it will so function, or will have such a functioning time. Thus the method of accumulating test data and that of analyzing them both differ from our ordinary methods.

In this report great pains have been taken to describe the operation of the new technique as it is applied to experimental designs, analysis of the data, and application of the results to the writing of meaningful specifications and the adoption of reliable acceptance sampling plans.

The initiator response of certain wire and carbon bridge detonators is described in terms of two characteristics which, by their joint action, make up a descriptive model of the initiator. The first one is functioning time. The manner of its variation with size and nature of input is first determined. From these data it is shown to be possible to evaluate functioning time graphically. Finally there is given the more precise analytical method of evaluation from the same data.

A similar technique is applied to the attribute of sensitivity. This requires the use of a new experimental design, and the advantages and disadvantages of the probit, the Bruceton, and Bartlett's designs are discussed and illustrated. Analysis of sensitivity data is discussed from the view point of the probit analysis, Berkson's logit analysis, the Bruceton analysis, a modified probit and logit analysis, and finally the analysis of Bartlett's design. Both graphical and analytical evaluation of the results are made. Finally, some of the possible applications that can be made of the information gathered from the analyses are stressed.

A comparison of the results obtained with a large number of detonators brought to light two points of interest: (1) manufacturer's lots of supposedly the same type of detonator differ enormously from lot to lot. No two lots are really alike. (2) These differences are often larger than the difference between different types of detonator.

There appeared to be a probable correlation between functioning time and functioning probability which, if it proves to exist, will simplify the testing of detonators.

1. INTRODUCTION

This report provides some of the much needed statistical background for the analysis of test data on both carbon and wire bridge initiators. Measurable parameters are developed that describe the response of the "average" initiator under various input conditions. The particular data presented in this report refer to various types of initiator and are used only to illustrate typical analyses. The methods described here are perfectly general, but require some adaptation for each initiators. In addition, it has become evident that new tools are frequently needed and should be developed in connection with any test studies made under this program.

The parameters obtained from the analyses lead to test levels suitable for specification purposes and acceptance sampling plans. This is important since quality control of so delicate a product as initiators can hardly be achieved without the use of statistics.

2. INITIATOR RESPONSE

Both carbon and wire bridge initiators have been tested extensively. The variables that influence their response are chiefly variations in the energy input from various sources, as well as temperature cycling and other forms of "conditioning." For example, when initiators are fired from a condenser the voltage of the firing source and the capacitance can be varied. Or, when they are fired from a constant current device, the magnitude of the current and its duration can be varied. Again, when initiators are fired from a constant voltage device, the level of the voltage and the duration of the impulse can be varied. Finally, when they are fired from an actual installation, a combination of many effects must be considered before the response of the initiators can be well understood.

It is true that a so-called thousand-erg detonator may be expected to respond with a probability of 0.999 (99.9 per cent of the time) if the energy source supplies 1000 ergs. However, this evergy package is not always sufficient to produce detonation, especially in a carbon bridge detonator. The firing capacitance must be of the right size and energy must be fed into the initiator at a rate which depends upon that size. Hence, high voltages are required for small capacitances, while large capacitances need only relatively small voltages. Variations in the firing voltage, therefore, affect the response of the initiator like variations in firing energy and capacitance combined.

It is frequently required that each initiator in a given lot react within a specified minimum time, or that the reaction time from one to another in the same lot vary as little as possible. For example, the specified maximum functioning time for the T18E3 carbon bridge detonator is 10 microseconds. Other less sensitive initiators have a larger functioning time. Nevertheless, the functioning time is not an exactly reproducible variable because of small variations in the construction of the detonators.

In summary, the functioning of initiators is now measured by an attribute and a variable, namely the quantal response if functioning and the resultant functioning time. The over-all response depends upon the input conditions. For large inputs, the initiator will almost certainly function with a short functioning time. But for smaller inputs, not all initiators will function, and those that do, will show relatively longer functioning times. Some of the initiators will even fail to function. This phenomenon is of the type that statisticians refer to as a yes-no or quantal response. Of necessity, it enters into the picture that we wish to paint of the over-all response of initiators. Therefore, we shall ultimately describe initiator response in terms of two characteristics: (1) Functioning Time (a variable), (2) Sensitivity (an attribute).

The joint action of these characteristics makes up a descriptive model of the initiator. For purposes of illustration only, we have chosen here various wire and carbon bridge detonators. The interaction of the characteristics separates the response into regions of applicability and non-applicability. From these regions it can be inferred in which application the use of any particular detonator is advisable or not advisable. The model also permits us to locate suitable levels for specification testing, acceptance sampling plans, and production quality control.

3. FUNCTIONING TIME

The functioning time of initiators, tested with the FILITS equipment, is measured with a complex instrumentation described elsewhere (Operation and Maintenance Manual, FILITS; Oct. 15, 1954). A point of interest here is the sensitivity of the apparatus. The time measurements are made with a counter chronograph registering units of one-eighth microsecond. Therefore, readings of about one microsecond and over are fairly accurate, since they are far from the possible instrumental error of $\pm$ 1/8 microsecond. Readings much larger can be considered free of error for all practical purposes. The source of energy which causes the deton ator to function-or fail, as the case may be-is a charged capacitor. It is charged to a predetermined voltage from a constant voltage source and is discharged through a mercury switch relay.

We must expect that the functioning time of initiators will vary both with energy or voltage imputs and with changes in the firing capacitance. To establish the magnitude of such variations, experiments can be made over the whole range of available voltages and capacitances. The resulting data show, then, the dependence of the functioning time, t, upon a source capacitance, C, applied voltage V, and energy parcel, W. An important factor in these tests is the fact that certain combinations of C, V, and W result in very low functioning probabilities. Therefore, the majority of tests is performed in a region where detonation is almost certain to occur. Tests in the fifty per cent probability firing region are available from stratified data obtained during sensitivity analyses (cf. Section 4). These data yield information even at the lowest probability levels. Tests outside of the mean functioning probability range are usually conducted only for inputs that assure a very high functioning probability. Such tests yield information under conditions where the functioning times are, in general, reproducible. The sum of all this information furnishes a descriptive picture of the variation in functioning time of a detonator together with the uncertainty due to its probability factors.

3.1 Constant Input Conditions

The first problem to be considered is the representation of functioning time data obtained under constant input conditions. As stated earlier, it is obvious that a test series performed with a certain detonator, even under constant test conditions, will result in a series of functioning times that are likely to differ from one another. Therefore, the description of such test results must be made in statistical language.

Table (3-1) shows test data obtained for T24E1 (AAP-20-1) detonators fired at 15.85 volts from an 0.100 microfarad condenser. The functioning times vary between 22 and 50 microseconds. There are two forms of analyses to which we can subject these data. In one form the average functioning time, $\bar{t}$, and its standard deviation (corrected for sample size) is computed directly from the observed data. In the other form the possible skewing of the data is taken into account and the work is carried out with the logarithms of the functioning times. The following equations can then be used:

$$
\begin{aligned}
t &= \text{observed functioning time, microseconds} \\
\tau &= \log t \qquad (1) \\
n &= \text{number of observations}
\end{aligned}
$$

$$\bar{t} = \Sigma t / n \qquad (2)$$

= average functioning time from raw data

$$s_t = \sqrt{(n\Sigma t^2 - (\Sigma t)^2) / (n(n-1))} \qquad (3)$$

= standard deviation in original units from raw data

$$\bar{\tau} = \Sigma \tau / n \qquad (4)$$

= average logarithmic functioning time

$$s_\tau = \sqrt{(n\Sigma \tau^2 - (\Sigma \tau)^2) / (n(n-1))} \qquad (5)$$

= standard deviation in logarithmic units from logarithms of functioning times

The connection between these formulas is not an absolute one. However, for nicely behaved data, the following approximations hold:

$$\bar{t} \approx \text{antilog}\,\bar{\tau} \qquad (6)$$

$$s_t \approx (\text{antilog}\,\bar{\tau})(\text{antilog}\,s_\tau - 1) \qquad (7)$$

The data of Table 3-1 are somewhat skewed and these two relations are, therefore, only approximately true. The reason for applying logarithmic transformations is that the distribution of the absolute functioning times is frequently skewed. The use of logarithms reduces the long tail for large functioning times and extends the part of the distribution that lies to the left of the mode. This is a fairly well known technique usually applied when only empirical data are available. It would naturally

Table 3-1. FUNCTIONING TIMES FOR 28 DAY-TEMPERATURE CYCLED DETONATOR T24E1 (AAP 20-1)

t, microseconds	τ
26.875	1.42935
44.625	1.64958
39.125	1.59246
50.25	1.70114
26.125	1.41706
48.25	1.68350
38.625	1.58686
39.625	1.59796
49.125	1.69130
45.375	1.65682
22.75	1.35698
31.125	1.49311

n	=	12	n	=	12
Σt	=	461.875	$\Sigma \tau$	=	18.85612
Σt^2	=	18800.578125	$\Sigma \tau^2$	=	29.78411
$\bar{t}$	=	38.5	$\bar{\tau}$	=	1.57134
s_t	=	9.6	s_τ	=	.11858

Antilog $\bar{\tau}$ = 37.3

(Antilog $\bar{\tau}$)(Antilog s_τ-1) = 11.7

$C = 0.1\mu f$

$V = 15.85V$

Ref: Data Page 345

be much more desirable to know something about how these functioning times ought to be distributed, but in the absence of such information simple, empirical methods must be used.

Either of the two methods will, therefore, yield a mean functioning time and some measure of the distribution about this mean, for a group of observations obtained under constant input conditions. What happens to the functioning time when the input conditions are varied will be discussed in the following section (3.2). Although the logarithmic treatment of the functioning time data involves a considerably greater amount of work than the analysis of the untransformed data, it is to be preferred wherever the data are highly skewed to the left, e.g., when they show a long tail for large functioning times. For less critical analyses, however. the use of formulas (2) and (3) yields the desirable parameters with greater facility.

3.2 Effects of Variations in Input

It is well known that the functioning time of a detonator is very dependent upon the type and the magnitude of the energy input provided it is sufficient to cause detonation. Since we have developed methods for analyzing functioning time data under constant input conditions in the preceding section (3.1) we can now apply these methods to any group of data obtained under any input conditions. Thus, the mean functioning times can be computed in tabular form for various input voltages and capacitances, as is shown for the T18E3 (Atlas Lot AAP 50-2), in tables 3-2 and 3-3. The first shows the mean functioning times obtained from equation (2), while the other table shows the standard deviations obtained from equation (3).

The tables serve to substantiate several experimentally observed facts of interest. First of all, the functioning time varies significantly (in the statistical sense) with the input. Appropriate significance tests have been performed to establish this fact beyond any doubt. For this particular detonator, we were able to obtain a mean functioning time as low as 2.8 microseconds for a high voltage input from a large capacitance. Such a functioning time is considerably below that required by specifications now set at 10 microseconds or less. On the other hand, a functioning time greater than permissible by specifications occurs for a low voltage from a medium or a large size capacitance, especially in the region where the functioning of the detonator is not very probable.

Second, the variability dispersion in the functioning time is also dependent upon the input. Table 3-3 shows that the standard deviation may be very small, which may indicate that the detonation, if set off with enough electrical "brute force" does not depend very heavily upon the mechanical and chemical variation in detonator construction. A detonation initiated near the fifty per cent firing probability level, on the other hand, shows great variability in functioning, namely, the standard deviation is both large and erratic. For example, for C = 0.01 microfarads and V = 50.2 volts, we find s_t = 5.6 microseconds; while for the same capacitance and V = 79.5 volts we have s_t = 0.5 microseconds. Significance tests have established that many of these variations in the estimated standard deviation are "real" in a statistical sense. This may indicate that a detonator, when not set off by electrical "brute force", tends to respond much more to the variability in construction by exhibiting a greater variation in the functioning time.

Finally, we may investigate the amount of energy required for detonation. Usually, the energy is computed from

$$W = 5\ C\ V^2 \tag{8}$$

where W is measured in ergs, while C and V are measured in microfarads and volts, as before. Table 3-4 shows the values of W, for a detonation within the stated range of input. For a constant capacitance and an increasing voltage, the energy fed into the detonator also increases. The table shows, therefore, the minimum energy required to obtain detonation.

Table 3-2. OBSERVED MEAN FUNCTIONING TIMES ($\bar{t}$) FOR THE T18E3 DETONATOR

(ATLAS LOT AAP 50-2)

	Firing Capcity C, Microfarads												
Firing Voltage	0.000217	0.0005	0.001	0.00127	0.005	0.01	0.0196	0.05	0.1	0.215	0.5	1.0	Weighted Row Average *)
12.6											21.4	66.1	43.7
15.9											36.3		36.3
20.0									24.4		49.3	126.4	98.0
25.2								9.0	44.6	43.5	8.8		27.4
31.7							11.0	20.1	15.9				18.6
39.9						8.3	9.8	21.1					13.6
40.0									17.6		7.5	13.3	13.2
50.2						8.1	10.0						9.5
60.0									3.4		5.7	4.8	4.8
63.2					4.0	3.2	21.6						9.1
79.5					4.9	3.2							4.2
80.0									2.9		3.2	3.6	3.2
100			3.4	3.5	3.1	3.4					2.8	2.6	3.0
120					2.5			3.0			2.8		2.8
126		2.9	2.8	3.4									3.0
159		3.1	3.0										3.1
160			2.9			2.9			2.9			2.8	2.9
200	3.1	2.8	3.2										2.9
240	3.0	2.7			4.0			2.7			2.6		3.0
252	3.2	2.7											2.9
317													3.0
320			3.0			2.7			2.8			2.8	2.8
480		2.4			2.5			2.8			2.8		2.6
640			2.8			2.9							2.9
960		2.5			2.8								2.6

*) For weights, cf. Appendix E.

Table 3-3. OBSERVED STANDARD DEVIATION S_t (MICROSECONDS) FOR THE T18E3 CARBON BRIDGE DETONATOR (ATLAS LOT AAP 50-2)

	Firing Capacity C, Microfarads												
Firing Voltage	0.000217	0.0005	0.001	0.00197	0.005	0.01	0.0196	0.05	0.1	0.215	0.5	1.0	Weighted* Row Average
12.6											17.5	66.7	48.8
15.9											3.6		3.6
20.0									—		19.4	93.2	85.0
25.2								6.1	29.3	48.7	11.7		29.7
31.7							—	13.5	—				13.5
39.9						2.0	7.5	14.2					10.2
40.0									21.5		8.8	7.1	14.4
50.2						5.6	7.5						7.1
60.0									0.6		6.7	2.0	4.6
63.2					0.8	0.4	15.9						8.5
79.5					2.9	0.5							2.4
80.0									0.4		1.4	2.2	1.5
100			—	0.1	0.1	—					0.5	0.2	0.3
120					0.2			0.4			0.4		0.4
126		0.2	0.3	0.3									0.3
159		0.2	0.4										0.3
160			0.4						0.6			0.2	0.4
200	—	0.3	—										0.3
240		0.4			3.4			0.1			0.3		1.7
252	0.2	0.3											0.2
317	0.1												0.1
320			0.2			0.3			0.1			0.2	0.2
480		0.2			0.4			0.4			0.5		0.4
640			0.3			0.2							0.3
960		0.5			0.2								0.4

*) For weights, cf. Appendix E.

Table 3-4. ENERGY INPUT W (ERGS) NECESSARY FOR FIRING OF THE T18E3 DETONATOR

(ATLAS LOT AAP 50-2)

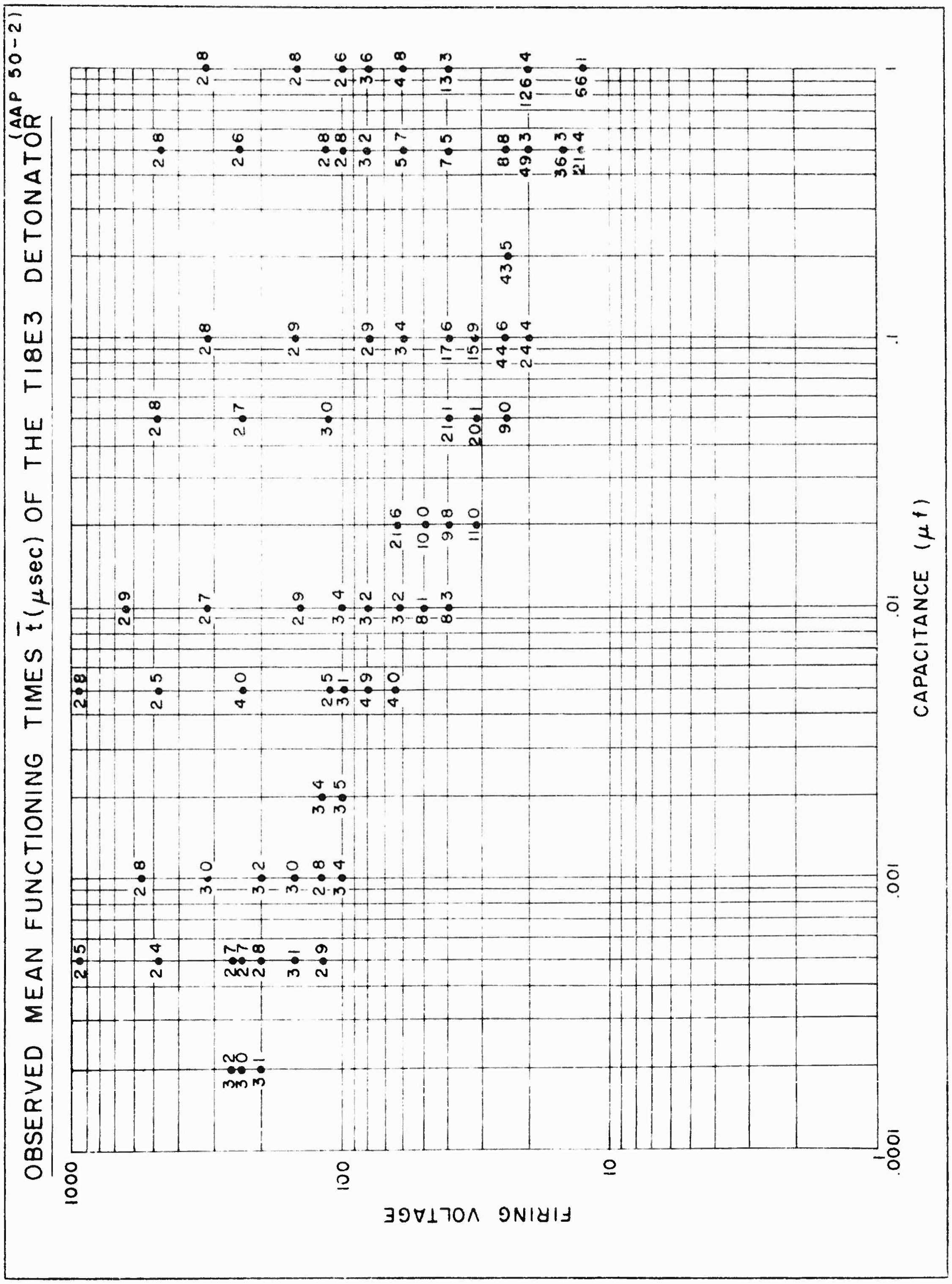
Firing Voltage	Firing Capacity C, Microfarads											
	0.000217	0.0005	0.001	0.00197	0.005	0.01	0.0196	0.05	0.1	0.215	0.5	1.0
12.6											397	794
15.9											632	
20.0									200		1000	2000
25.2								159	318	683	1588	
31.7							98	251	502			
39.9						80	156	398				
40.0									800		4000	8000
50.2						126	247					
60.0									1800		9000	18000
63.2					100	200	391					
79.5					158	316						
80.0									3200		16000	32000
100			50	98	250	500					25000	50000
120					360			3600			36000	
126		40	79	156								
159		63	126									
160			128			1280			12800			128000
200	43	100	200									
240		144			1440			14400			144000	
252	69	159										
317	109											
320			512			5120			51200			512000
480		576			5760			57600			576000	
640			2048			20480						
960		2304			23040							

The mean functioning time and the variability in functioning time may be considered dependent upon input stated in terms of voltage and capacitance, or in terms of energy. The T18E3 type detonator is seen to be responsive to an energy input as small as 40 ergs in one instance. But we note also that for a large capacitance and a small voltage a much higher energy input is required to insure detonation. This fact causes us to question the advisability of the frequently used terminology "100-erg detonator" or "1000-erg detonator". This terminology has little, if any validity, in the light of the fact that a statement of energy alone is insufficient to characterize the response of a detonator.

3.3 Graphical Evaluation of Functioning Time Data

Since functioning time is a characteristic property of all detonators, the user frequently needs to predict the functioning times of a particular detonator for some projected installation. If we return to Table 3-2, we find that we can be certain that the T18E3 detonator will have a functioning time less than 10 microseconds, if it is fired from a 0.01 microfarad condenser with a voltage exceeding 60 volts. But the question arises, whether we cannot state the functioning time more accurately by shaving it down a bit. To do so we must use a probability model incorporating unfamiliar and technical terms. We shall now try to develop such a model and show how its parameters can be found graphically.

We shall first arrange the observed mean functioning times in a plot, such as that shown in Figure 3-1. This plot shows the observed mean functioning times in microseconds at those points, with coordinates (log C, log V), where they have been observed. The observed points are samples taken on the hypothetical model $\bar{t} = \bar{t}(C,V)$ which represents a surface in three-dimensions. Because of the variation in the construction of a detonator, sampling cannot possibly yield the true surface at most points. It is necessary to find an approximation to the true surface by means of smoothing. In the case of a detonator, the smoothing amounts to the search for a family of curves $\bar{t} = \bar{t}(C,V)$ which come close to the



REPORT I-1804-1 FIGURE 3-1

observed points. The family is naturally not constructed for the type of average functioning times observed, e.g., 2.8, 2.9, 3.4, etc. microseconds. The family is rather constructed for easily assayable functioning times such as 3, 4, 5, 10, 20, etc. microseconds and it is assumed that the reader can find other mean functioning times by interpolation.

With some experience in graphical interpolation, we can readily construct such a family of curves. In actual practice, it is advisable that several curves be constructed independently and that the final smoothing be done by means of a graphical averaging process. It is essential that the type of model for the construction of the family of curves be chosen to conform with the empirical data. In table 3-2 we computed row averages of the mean functioning times (right hand column) as if the mean functioning times observed were possibly independent of the firing capacitance. Some of the earlier graphs released under this project were actually based upon such a model. However more recent studies indicate that this model holds only within a narrow range. If we assume that mean functioning time is independent of firing capacitance, the model yields "horizontal" lines for constant functioning times, as shown in Figure 3-2. If we do not assume such independence, the model yields curved lines for constant functioning time, as shown in Figure 3-3. The latter is without doubt the better representation of the observed functioning times and their respective averages.

We have described this graphical smoothing procedure for the mean functioning times in great detail because the procedure as such is generally applicable to other detonator data. The next step is to use a similar procedure to incorporate the observed standard deviations into the picture of the functioning times. It is sufficient to describe it with less detail and to exhibit only the final results.

We have prepared a plot of the observed standard deviations that is similar to that of the mean functioning times in Figure 3-1. From this plot we readily obtained another, showing multiples of the smoothed

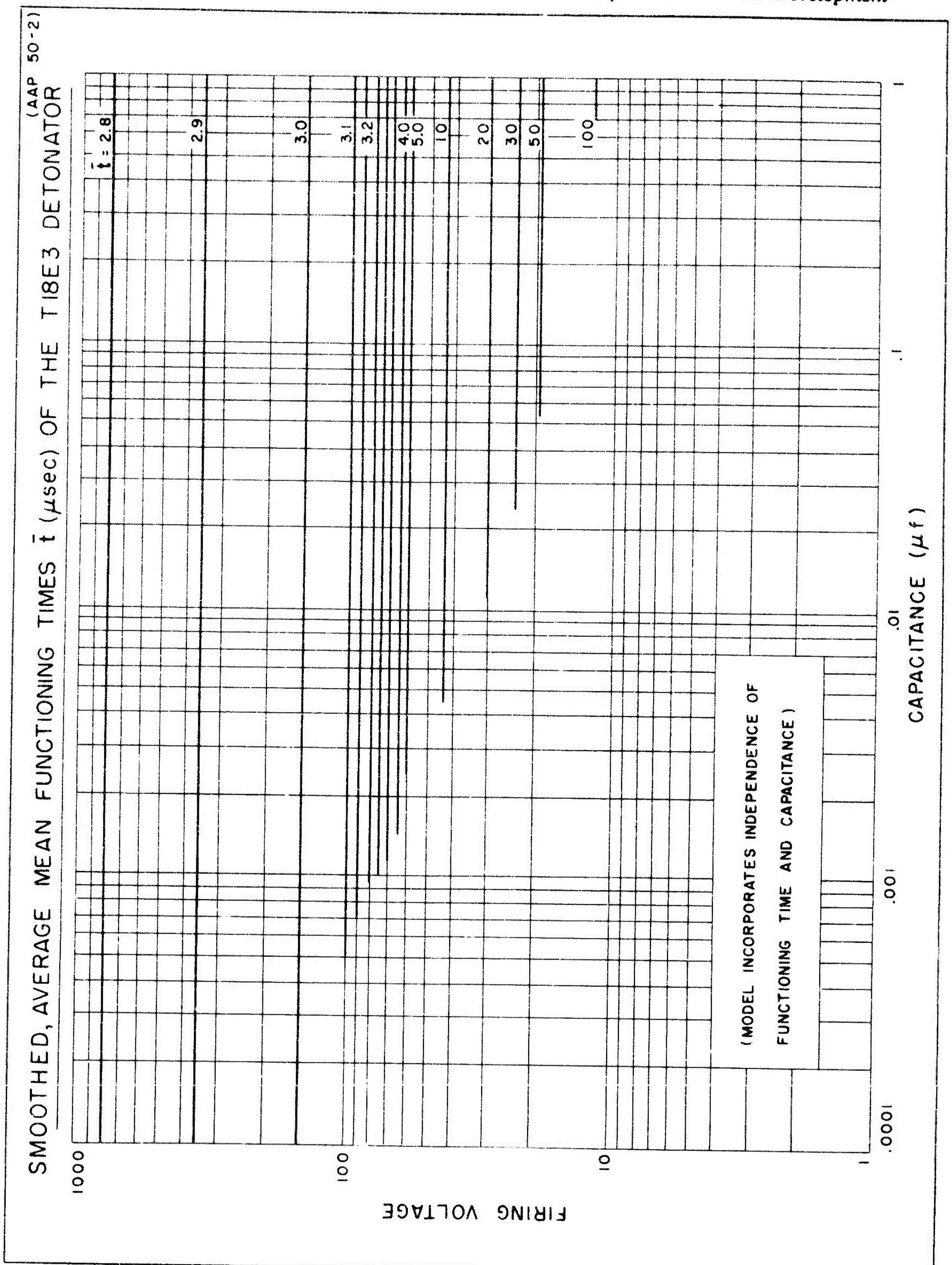


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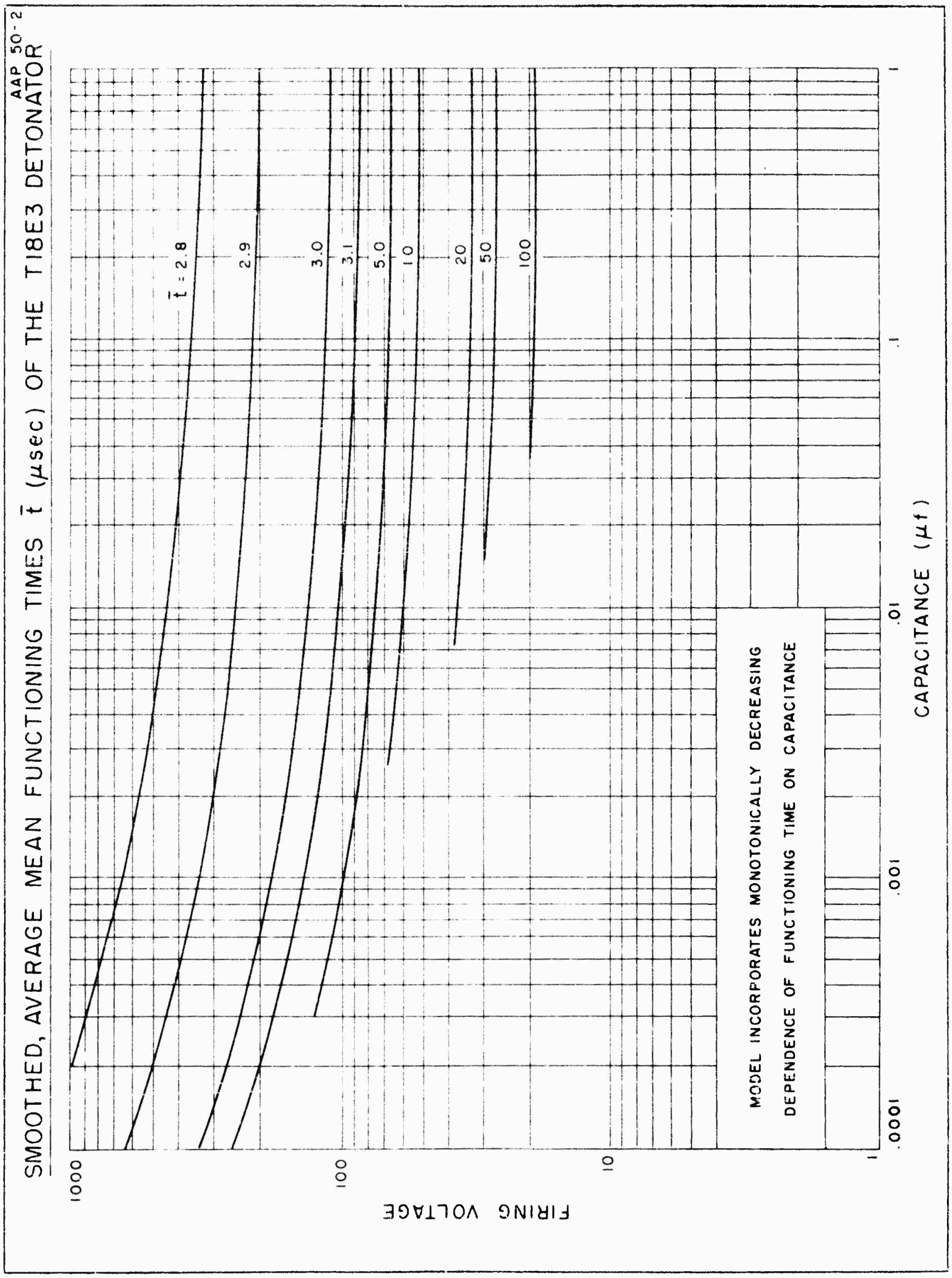
FIGURE 3-2



REPORT I-1804-1 FIGURE 3-3

standard deviations. Useful multiples are the well known terms 1.645 s_t, 2.327 s_t, and 3.09 s_t which refer to the 95%, 99% and 99.9% probability levels respectively, on the assumption that there is a normal distribution of the data. By adding the levels obtained from the multiple standard deviations and those obtained earlier for the mean functioning times, we can obtain estimates of the upper limits of the functioning times which correspond to certain given probabilities. Figure 3-4 shows the 99% and 99.9% levels obtained in this manner for the T18E3 (AAP 50-2) carbon bridge detonator.

In order to use this diagram let us assume that in a given installation with this particular detonator a firing capacitance of 0.01 microfarads and a firing voltage of 200 volts are available. We can then state that on the average this particular detonator will have a functioning time of 2.95 microseconds. We can further state, with a probability of 99.9%, that under these conditions the detonator will function in less than 5.3 microseconds.

This chart may also be used as follows: Given a firing capacitance of 0.1 microfarads, what firing voltage must be available so that we can "guarantee" a functioning time of less than 10 microseconds? Referring to the chart, we find that $t_{.999} = 10$ and $C = 0.1$ intersect at $V = 80$ volts. Therefore, with such a firing voltage we can be almost sure that the detonator under test will have a functioning time less than 10 microseconds.

Concerning the data necessary for the construction of a model of detonator functioning time, we return to Figure 3-1. We note in passing that not all points in this diagram were obtained with the same precision: some represent samples of 5, others as many as 20 detonators. The reasons for such unequal sample size in the several points is that they do not all stem from the same source in the testing program. The "lower" points in the diagram are obtained throughout during the sensitivity tests; only the "upper" points are obtained from specially designed

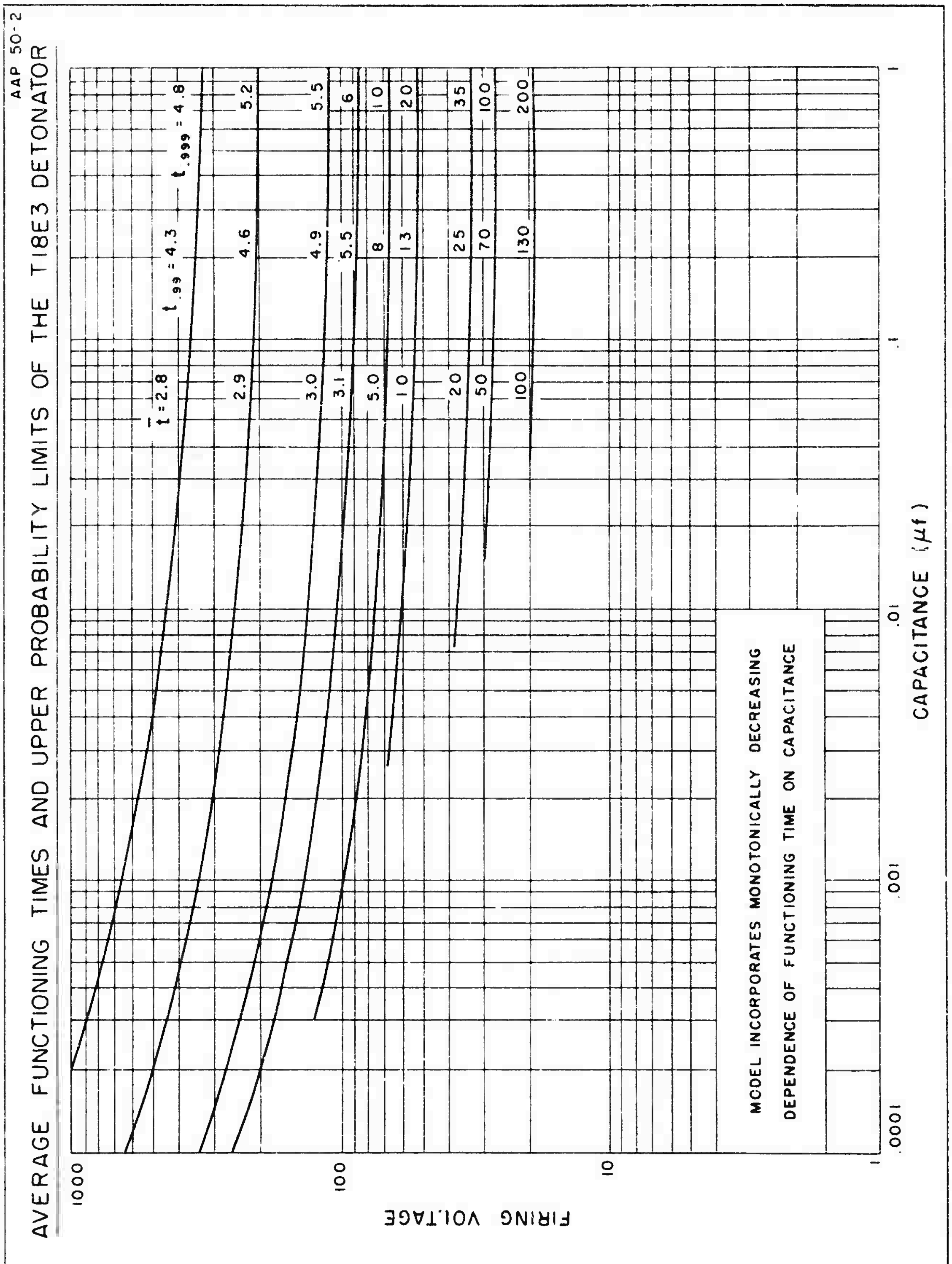


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FIGURE 3-4

functioning time tests. Figure 3-1 shows that the experimental procedure will give good coverage of the functioning time area if we adopt a checker-board design which produces enough points, distributed fairly uniformly, so that the family of curves can be constructed. Such a procedure increases the number of units to be tested. How many detonators should be tested for functioning time at each point? Since it is not necessary to obtain the functioning time with great accuracy samples of six at each point are sufficient. If the units are expensive the sample size may have to be reduced, as well as the number of test points covering the area in Figure 3-2.

Finally, how accurately does the family of curves obtained from this graphical procedure describe a given detonator? This question cannot be answered definitively; however, there are available some statistical tools that give the required answers in terms of probability. The next section of this report will be devoted to such analytical studies and will report on functioning time evaluations made both graphically and analytically.

3.4 Analytical Evaluation of Functioning Time Data

After the functioning times for various combinations of firing capacitance, C, and firing voltage, V, have been determined, we may need a more refined model for the detonator under test than that described in connection with graphical smoothing procedures (Section 3.3). The model will now take the form of a mathematical equation. Since nothing is known about the "true" model, an approximate equation must be chosen that may represent the data adequately. Therefore, experience gained with the graphical procedures will be very helpful.

Returning to Figure 3-4, we find that the model does take the form $t = t(V, C)$, but that it will be more convenient to deal with the logarithmic model instead: $\tau = \tau(\log V, \log C)$. The shape of the curves obtained thus far indicates clearly that this function will contain non-linear terms as well as interdependent terms between log V and log C. Therefore, it may well be represented in the form of a complete Taylor expansion for two variables:

$$\log t = \tau = a_{00} + a_{10} \log V + a_{01} \log C + a_{20} \log^2 V + a_{11} \log V \log C + a_{02} \log^2 C + a_{30} \log^3 V + a_{21} \log^2 V \log C + a_{12} \log V \log^2 C + a_{03} \log^3 C + \ldots \quad (9)$$

Such an expansion makes good sense in the region in which we study the functioning time phenomenon. There are no singularities in this region and the function can be expected to be a well-behaved one. It is now merely a question of obtaining the parameters a_{ij} by an appropriate statistical procedure and of learning whether the introduction of additional parameters a_{ij} yields a better fit to the empirical data.

The procedure is as follows. We must find the function that fits the given data best. Let us try the first term only in Equation (9).

$$\tau_1 = {}_1a_{00} \quad (10.1)$$

This function or model states that the functioning time is independent of voltage and capacitance and that all observed variations are merely due to chance. Evidently, we know better; but the function will be calculated in order to yield what statisticians refer to as a null-hypothesis. Other functions will be compared with it. The differences between observed values and those computed from this function are apt to be large, since the function will not fit the data well. The sum of the squares of these differences thus yields a large "variance" which can be tested against the smaller variances obtained from later models.

The next step in refinement of the model is to try a function consisting of the first three terms of Equation (9).

$$\tau_2 = {}_2a_{00} + {}_2a_{10} \log V + {}_2a_{01} \log C \qquad (10.2)$$

This model implies that the logarithm of the functioning time depends linearly upon the logarithms of the firing voltage and of the firing capacity, respectively. If we inspect Figure 3-4 once more, we come at once to the conclusion that this model may possibly be more realistic than (10.1), but that it still deviates largely from the "true" and, unfortunately, unknown model. However, we may proceed to estimate the three parameters which appear in (10.2) from the given data and to compute the resultant variance. This variance should be smaller than that obtained from (10.1) and the variance reduction is a measure of how significantly we have improved our model.

Let us complicate our function still further by the addition of three more terms from Equation (9).

$$\tau_3 = {}_3a_{00} + {}_3a_{10} \log V + {}_3a_{01} \log C + {}_3a_{20} \log^2 V + {}_3a_{11} \log V \log C$$
$$+ {}_3a_{02} \log^2 C \qquad (10.3)$$

This model implies non-linear dependence as well as interdependence between the logarithms of firing voltage and firing capacity. It leads to a certain variance, which is necessarily smaller than the preceding variance. The variance reduction is again a measure of how significantly the model has been improved by the introduction of new parameters.

This process may be continued with more and more parameters. However, the computation soon becomes very unwieldy. The mathematical-statistical details of the least squares fitting to the data are shown

Table 3-5. SUMMARY OF STATISTICAL PARAMETERS FOR VARIOUS FUNCTIONING TIME MODELS OF THE T18E4 (R < 1000 Ω) CARBON BRIDGE DETONATOR

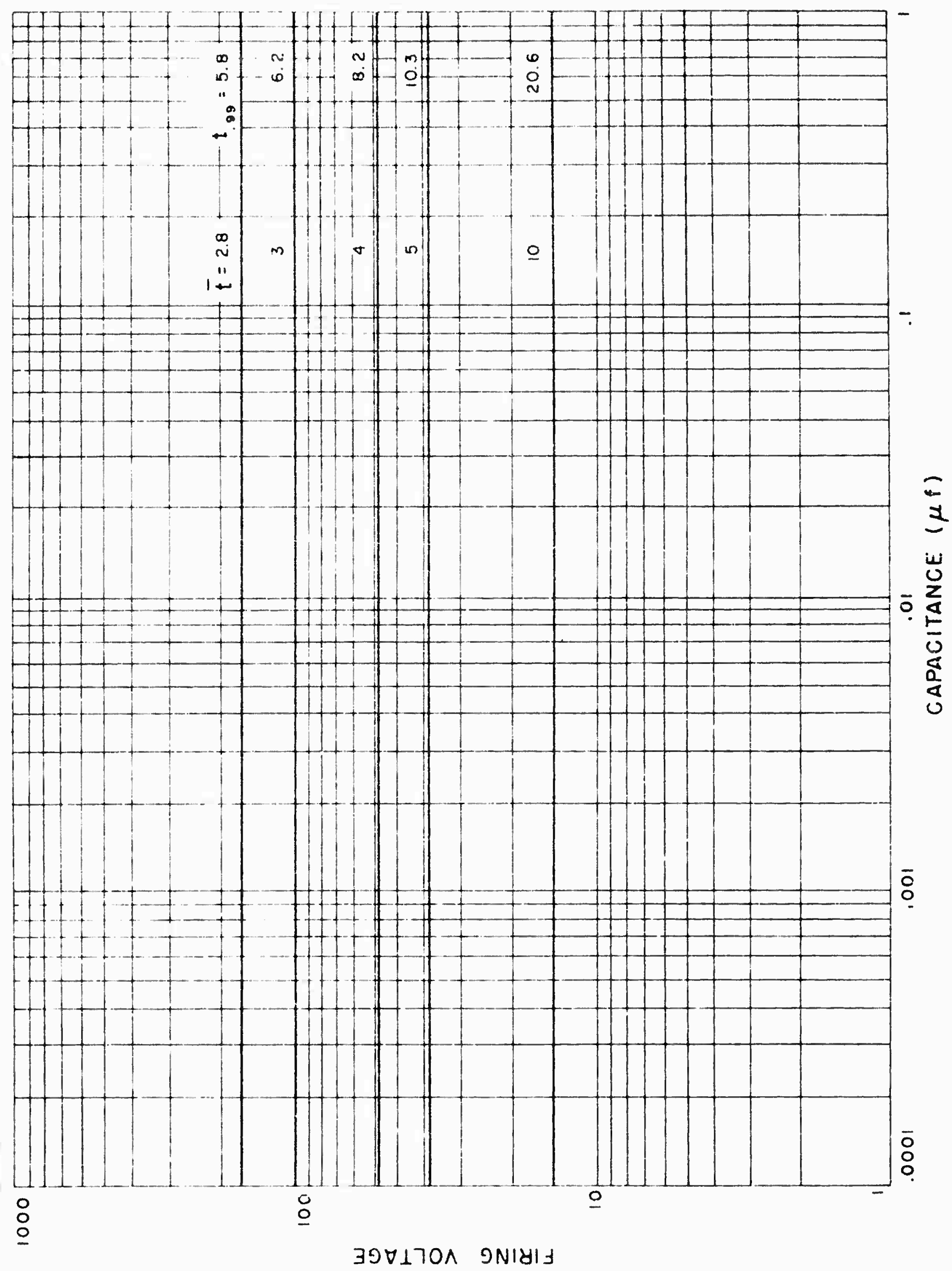
Model	Parameter Values	Variance	Degrees of Freedom
τ_1	${}_1a_{00}$ 0.68231	0.07918	152
τ_3	${}_3a_{00}$ 2.52248	0.01807	150
	${}_3a_{10}$ -1.69525		
	${}_3a_{20}$ 0.34298		
τ_5	${}_5a_{00}$ 2.37508	0.01723	148
	${}_5a_{10}$ -1.44256		
	${}_5a_{01}$ 0.12950		
	${}_5a_{11}$ -0.05244		
	${}_5a_{02}$ 0.26839		
τ_6	${}_6a_{00}$ 1.27757	0.02659	147
	${}_6a_{01}$ 0.60725		
	${}_6a_{20}$ -0.09918		
	${}_6a_{11}$ -0.15031		
	${}_6a_{02}$ 0.12624		
	${}_6a_{03}$ 0.01983		
τ_7	${}_7a_{00}$ 2.62069	0.01371	146
	${}_7a_{10}$ -1.66477		
	${}_7a_{01}$ 0.41713		
	${}_7a_{20}$ 0.31959		
	${}_7a_{11}$ -0.02738		
	${}_7a_{02}$ 0.22454		
	${}_7a_{03}$ 0.03440		

in Appendix A to which we refer the reader who wishes to carry out such a computation for himself. We are here most interested in the results of computations for the T18E4 (R < K : special) carbon bridge detonators. The reasons for this choice were mainly that at the time the calculations were prepared these particular data were available in tabulated form, while other data were not available. Table 3-5 shows the values of the parameters for the four different models tried, as well as the result obtained for the null-hypothesis. The models do not quite correspond to the equations stated earlier. Some of the parameters were omitted, as graphical analysis showed them to be of insignificant value. The variance reduction for the various models is impressive. It also indicates that the most complicated model, τ_7 with seven parameters, could still be improved. However, lack of computational time prevented us from developing the model further.

Figures 3-5, 3-6, and 3-7 depict the models τ_3 , τ_5 , and τ_7 . The simplest model τ_3 assumes independence of the functioning time from firing capacitance, and non-linear (parabolic) dependence of the logarithmic functioning time on the logarithm of firing voltage. The resultant curves--straight lines--are "horizontal" lines by nature of the model. If we compare them with the models for τ_5 and τ_7, we can see why in some of the first graphically smoothed models we had assumed that the lines were "horizontal" rather than curved. This detonator does not differ much from others in that respect. Only a considerable amount of testing and future work can establish beyond any doubt which model describes the data more exactly.

Figure 3-8 shows the functioning times obtained from the graphical smoothing procedure, described in Section 3.3 of this report. A comparison of these diagrams shows that the graphical procedure yields acceptable results, while the analytical procedure enables us to determine more closely the underlying model.

The 99% probability limits on the functioning time $t_{.99}$ were



THREE PARAMETER MODEL τ_3 FOR THE T18E4 (R<1000Ω) FUNCTIONING TIMES (μsec)

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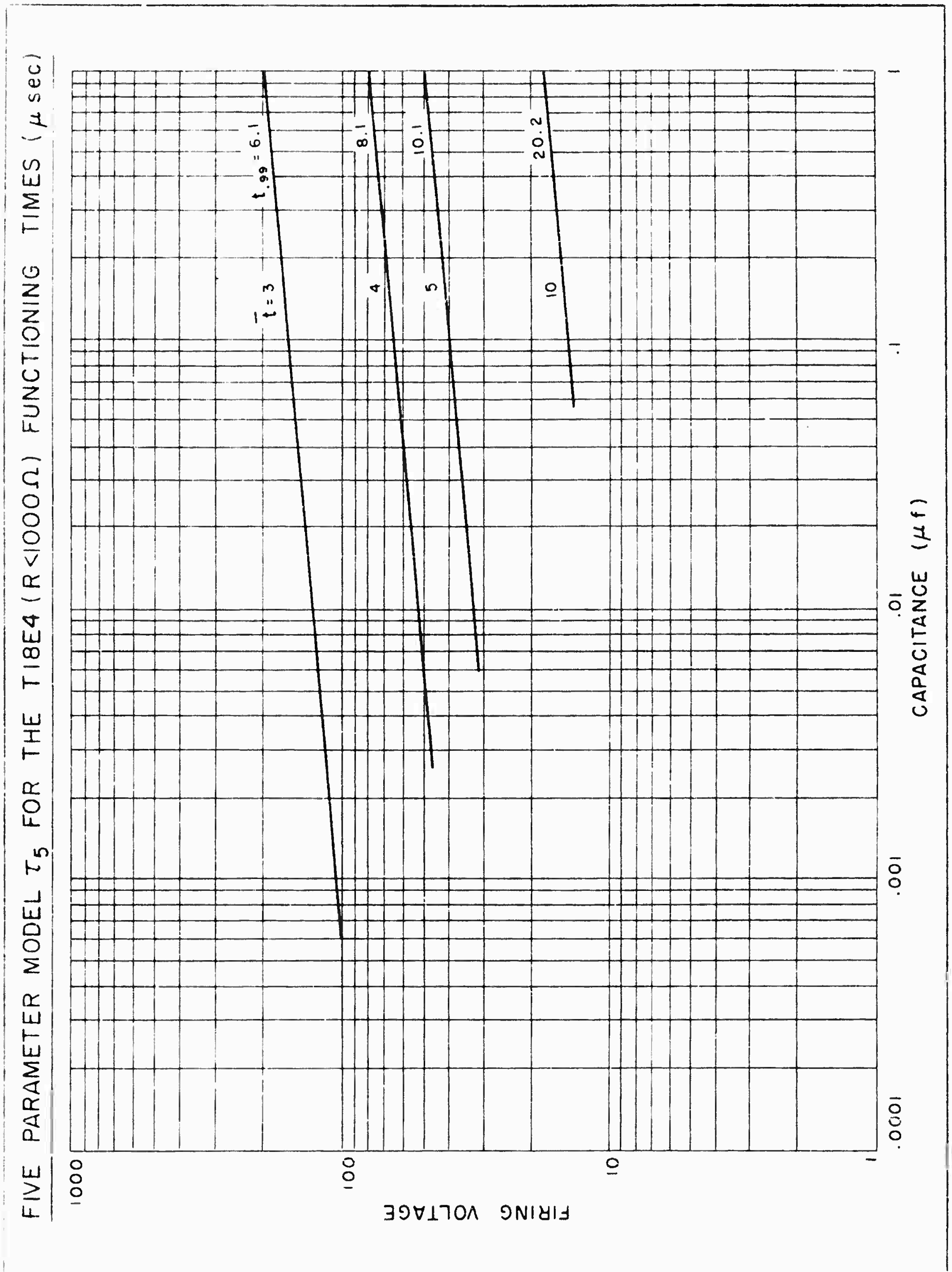
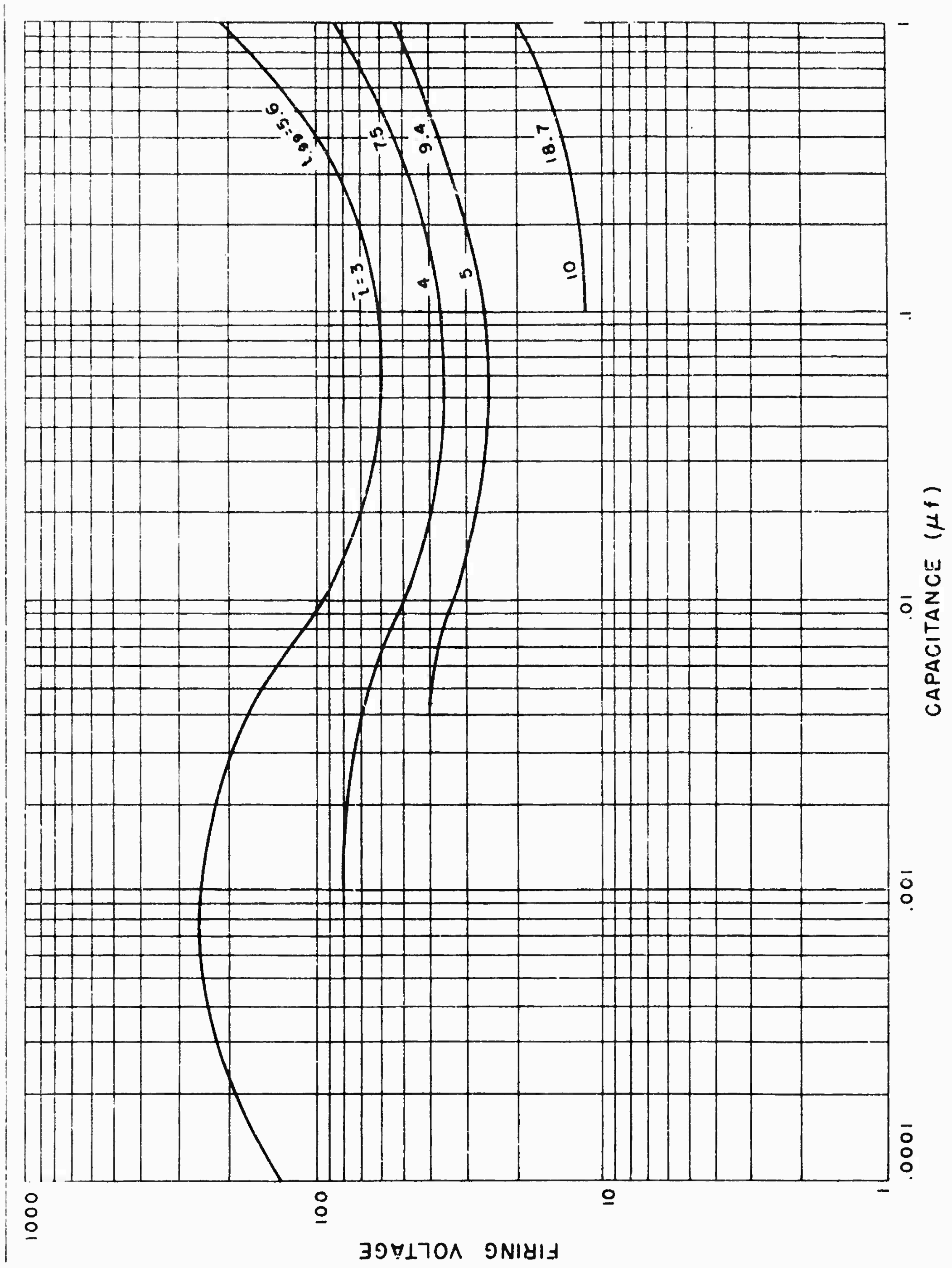
FIGURE 3-5



REPORT I-1804-1 FIGURE 3-6



SEVEN PARAMETER MODEL τ_7 FOR THE T1BE4 (R<1000Ω) FUNCTIONING TIMES (μ sec)

REPORT I-1804-1 FIGURE 3-7

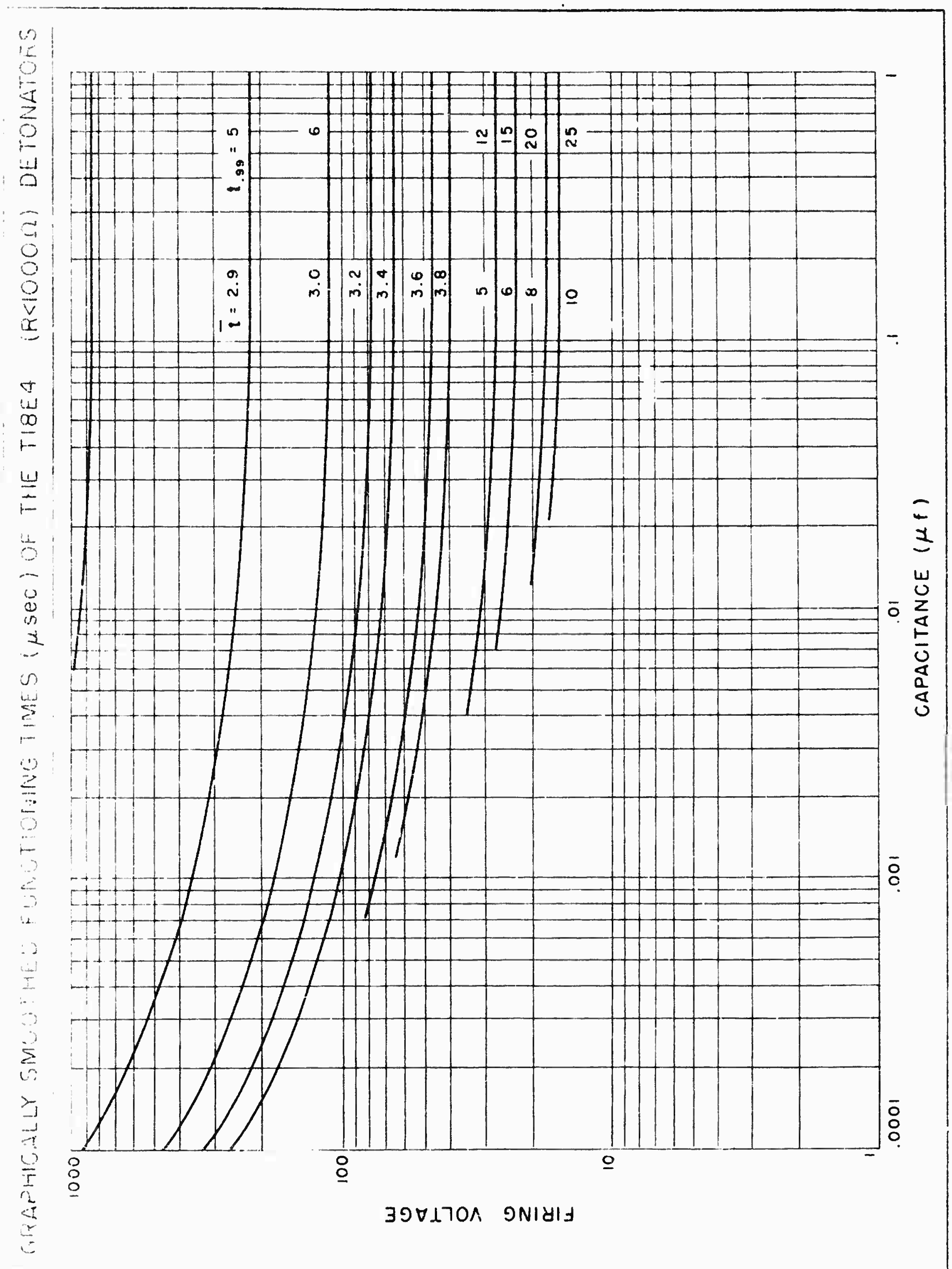


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FIGURE 3-8

obtained from equations (6) and (7) for the analytically smoothed graphs and the variances shown in Table 3-5. The graphically smoothed limits for $t_{.99}$ were obtained by the procedure described in the preceding section. The values compare favorably with the analytically obtained values, again indicating that in general the graphical procedure yields acceptable results, even though this procedure may not provide the best representation of the "true" functioning time model.

3.5 Resumé

The functioning time of a detonator is an observable, variable characteristic. Because of unavoidable variations in the manufacturing process of detonators, and effects not exactly controllable in the instrumentation, the functioning times observed under "constant" input conditions will vary from one detonator to the next. In order to obtain a graphical representation of the detonators' functioning times, a mathematical model has been developed which states that the functioning time is dependent on the (logarithm of the) firing voltage and the (logarithm of the) firing capacitance: $t = t(\log V, \log C)$.

The uncertainties arising from random variations in detonator construction and operation of the instrumentation can be lumped into one random variable which permits us to state that the observable functioning time will be less than some ascertainable quantity with a specified probability $1 - \alpha_P$:

$$P \left\{ t \leq t(\log V, \log C) + k_P s_t \right\} = 1 - \alpha_P \qquad (11)$$

The solution of this equation in terms of t_P leads to functioning time lines with constant probability

$$t_P = t_P(\log V, \log C, k_P, s_t) \quad . \qquad (12)$$

These lines can be obtained from a graphical procedure described in detail in Section 3.3, or they may be obtained from an involved analytical procedure (but with greater precision), described in Appendix A and Section 3.4 of this report. Either method yields a satisfactory description of the functioning times of a given detonator, provided sufficient data are available in the "area" in which the detonator will function with a probability exceeding 0.5. Several such diagrams are exhibited here in addition to the many diagrams published separately in other progress reports prepared under this project.

For the type of data usually collected during detonator evaluation, the application of graphical methods in the determination of the functioning time model seemspreferable--at least for the time being--to analytical methods. The graphical methods are speedy and yield results compatible with the desired precision. The analytical methods, although more precise, seem to require the use of a high-speed electronic digital computer for efficient application. Wherever such a machine is available, therefore, the analytical method should be given more attention than we could give it.

3.6 Selected References

Confidence Intervals

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3. C. W. Churchman, Statistical Manual, Methods of Making Experimental Inferences, Pitman-Dunn Laboratory, Frankford Arsenal, 1951.
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5. W. E. Deming, Statistical Adjustment of Data, John Wiley & Sons, 1943.

6. A. Hald, Statistical Theory with Engineering Applications, John Wiley & Sons, 1952.

7. R. M. McClung, First Aid for Pet Projects Injured in the Lab or on the Range, or What to do Until the Statistician Comes, Aviation Ordnance Department, NOTS Technical Memorandum No. 1113 China Lake, California, 21 January 1953.

Table for Tolerance Limit

8. A. H. Bowker, Tolerance Limits for Normal Distributions, Chapter 2, Techniques of Statistical Analysis, New York, 1947.

4. SENSITIVITY

A detonator may be set off by a package of energy supplied by a charged condenser. An extremely small amount of energy will not cause the initiator to respond; it fails. An extremely large amount of energy will set if off in all cases; it functions. Between these two extremes, a detonator will react only with a probability commensurate with the amount of energy supplied. This response of the detonator is a quantal response, of the yes-no type. Only the more important of the relevant contributions of statistical theory to this problem will be discussed here. The later subsections will deal with problems that arise from experimental design and analysis of data.

The detonators used in our tests come mostly from a production line. The line may be newly established, or it may have been in operation for a period of years. In any case statistical equilibrium in any subsequent analysis can be insured only if all samples are taken at random. This is a prime requisite in all sampling problems.

Sampling occurs twice in the evaluation of detonators. First, the units taken from the assembly line must be chosen in a random manner. Production may start up every Monday morning, hit a peak on Wednesday, and shut down on Friday noon. If so, the sample should be taken over the period of a week, or longer. This practice will remove any effect that the day of the week may have upon production. It is a necessary precaution since weekly operations show cyclic fluctuations in output quality. In many instances, there has been observed a slow rise in quality from Monday until Thursday, followed by a drop in quality on Friday.

Second, the random sample taken from the production line must again be sampled to perform tests that are independent of any strata in the sample. For example, the output of the production line may consist of cards, each containing five detonators. The cards are the strata from which the test units must be taken at random. To test one hundred units from a sample of one thousand, it is not proper to use twenty detonator cards of five units each, even if they were selected at random from the available 200 cards. Rather, the thousand units should be given cardinal numbers and selection should be made according to some scheme involving tables of random numbers. For example, we may identify each card and the position of the unit on any card by separate cardinals. Only random selection from both cards and positions assures the elimination of systematic deviations from the norm, possibly incurred during the packing process.

Experts in sampling theory and practice agree that random sampling is paramount to any success in the statistical analysis of data. This is especially true if the studies involve only part of the entire production. If the whole output could be tested, the problem of randomization would not be serious. But in destructive testing one must resort to sampling procedures, hence random selection is a "must."

4.1 Constant Input Conditions

Unlike the testing of detonators for a variable characteristic, such as functioning time or resistance, tests for a quantal response under constant input conditions provide little information. However, to understand the following forms of analysis more readily, we shall first develop some notions about testing under constant input conditions.

In the introduction to this section, we stated that a given type of detonator will certainly not function if the input is small enough. Likewise, the detonator will certainly function if the input is powerful enough. Between these two extremes lies a region about which little can be said a priori. If we knew exactly what each detonator unit would do under given input conditions, there would be no problem of testing and evaluation. However, assume for a moment that two levels are known at which we can be sure of function and malfunction, respectively. Each level may be characterized by a certain firing voltage from a given capacitance, or by a certain firing capacitance for a given voltage. It is evident that for a test performed at an intermediate level, no prediction can be made in absolute terms about the response of the detonator. All statements must be couched in the appropriate form of probability language. We shall then say that the probability that a given TYPE of detonator responds to a given input is a function of this input. If necessary, extensions involving other variables can readily be made. This statement in mathematical terms becomes

$$P_x = P_x(V, C) \tag{13}$$

where P_x indicates the probability of firing. The complementary probability for non-firing or failure is

$$P_o = 1 - P_x \tag{14}$$

These probabilities are subject to experimental determination. For a given lot of detonators, the probabilities could be established nicely for any desired combination of C and V by simply testing the entire lot on that (P,V)-level. However, since we deal here with destructive testing, it would be only of academic interest to learn that a lot of

1000 detonators tested with 200 volts at C = 0.001 microfarads resulted in 46 failures, while 954 detonators functioned.

However, there are certainly cases where testing at one level is of interest and value. We refer to acceptance testing in connection with a certain acceptance sampling plan. Specifications of a certain detonator may call for exactly such a test; for example, MIL-STD-105A requires that a certain number of units be tested at one level, and that no more than a specified number of failures be permitted. However, these specifications were established from the knowledge of the functioning probability at this level and we get into a vicious circle if we try to determine the level from acceptance sampling tests so as to obtain the acceptance sampling specifications.

It is necessary that we consider testing on various input levels, so as to obtain a measure of the functioning probability for a given detonator. The next section of this report will elaborate on methods that can be used effectively to gain information about the underlying probability curve which describes the functioning of a given detonator.

4.2 Effects of Variations in Inputs

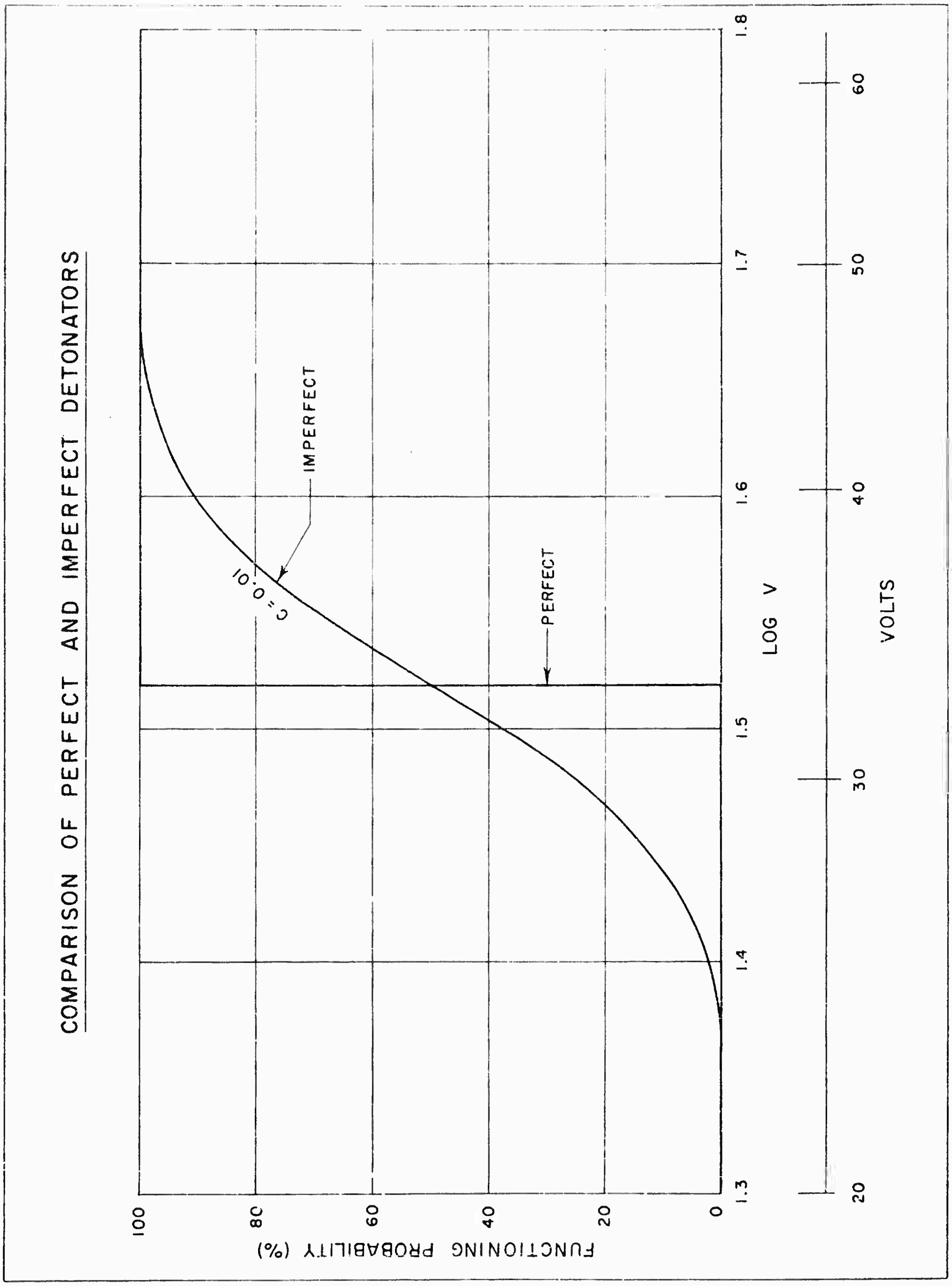
For the sake of convenience we shall confine our remarks to variations in the firing voltage. The results of these studies are easily translated into results that come from the variation of other parameters. From the discussion presented above, we see also that the response (e.g., the functioning or mal-functioning) of a detonator is characterized by some probability curve.

First of all, a "perfect" detonator has precisely reproducible characteristics. If a unit of a lot of perfect initiators detonates under a discharge of 33 volts from an 0.01-microfarad condenser, then all units of this type will function under such a discharge. Second, no unit will explode if we charge the condenser to only 32.99 volts if 33 volts is the characteristic lower detonation limit for this "perfect" initiator and the stated capacitance. Unfortunately, such a detonator has not yet been built, in fact, it is a far cry from the realization of this perfect detonator. In Figure 4-1 this non-existent "perfect" detonator is compared with its realized counterpart. We notice that the functioning probability of perfect units changes abruptly from zero to one hundred per cent. That is to say, such units exhibit a perfectly predictable performance. The response of imperfect detonators, on the other hand, is far from predictable in the classical sense. For example, the probability that a unit will explode at 25 volts is only 5 per cent. But that does not mean that if we take 20 units and test them at 25 volts, we can count on exactly one explosion. Nothing of the sort is implied. In order to verify the 5 per cent probability, within stated confidence limits, we might have to test hundreds or even thousands of units.

To the statistician, the curve of Figure 4-1 looks familiar. Similar curves are found in almost every textbook on statistics. Much ado has been made about the so-called normality of this curve, but there exist infinitely many curves of this type and there is little reason to expect that the "true" detonator curve is "normal." The only thing that can be stated about it is that it is a typical ogive, or S-shaped curve. Without considerable theoretical investigation we have no right to expect that it is normal, or Gaussian, and follows the Equation

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{(x - \bar{x})^2}{2\sigma^2}\right) dx.$$

Furthermore, since we have plotted the abscissa on a logarithmic scale, this distribution function has been normalized. But is also known that no transformation can normalize a non-normal distribution function <u>exactly,</u> especially in the region of the tails, and this is the region in which we are particularly interested. Therefore, we see that it is necessary to ferret out the parameters underlying this curve. The exact nature



REPORT I-1804-1 FIGURE 4-1

of the entire curve can never be established; however, the curve can be located within narrow intervals--stated in terms of probability -- by a suitable experimental procedure, known as experimental design. The next sections deal with several of the applicable designs.

4.3 Experimental Designs for Quantal Response Tests

Many studies have been made of the so-called quantal response tests. Various forms of experimental designs have been developed, permitting estimates of the desired parameters. These probability parameters must be found from the performance data of tests arranged according to some experimental design. The ultimate aim of all such tests is the description of the detonators in probabilistic language. We want to know--and this is typical of all quantal responses--what is the functioning probability of a given detonator at any given energy level. The higher the input level, the greater the probability that the detonator will respond without failure.

At least three designs have been used rather extensively. They are (a) Probit Design, (b) Bruceton Design, (c) Bartlett's Design. Each design measures certain parameters of the underlying probability distribution and tends to maximize the probability that the calculated parameters agree with the actual parameters.

The most efficient experimental design depends upon the true values of the parameters. If these parameters were known, the experiment would become unnecessary. Therefore, advance estimates of the parameters are usually made, or the experiment is designed around certain invariants. If both the mean and the standard deviation are unknown, the most efficient design would minimize the variances in both of these parameters. Unfortunately, this leads to a mathematical contradiction. A design which minimizes the variance of one parameter does not minimize the variance of the other, hence a compromise must be made.

4.3.1 The Probit Design

This method of testing quantal responses applies only to data with a normal distribution. The method gets its name from probability units, or probits, which are used in the analysis. A number of initiators are tested for yes-no response at various energy levels. These levels are usually chosen near the mean response level and at distances of one or two sigmas away from the mean. The test levels should vary randomly between the chosen limits. If 1, 2, 3, ... is the order of the units tested and a dashed number indicates negative response, an experiment performed with the probit design appears as in Figure 4-2. Sequential test levels are random and independent. The design yields a random process since (1) the results of any test depend only on the test level but not on the outcome of the preceding test, and (2) the results of any test do not affect the choice of the next test level.

The results of such an experiment are shown in Figure 4-3 in the form of a bar chart. The number of tests performed at each energy level is the same, but the fraction of positive responses increases as the test level goes up.

Table 4-1 contains some actual data obtained with T18E4 detonators and reported on our data sheets, pages 231, 238. We fired ten units at each level. The narrow spacing of the levels was chosen in order to obtain a good estimate of the mean firing level from the resultant experimental evaluation. The table indicates that the data embrace the mean firing level--which was found to be located near 136 volts--but they do not extend into the region where either misfires or fires become highly improbable. Analysis of the data will be discussed under the appropriate subsection (4.4.1).

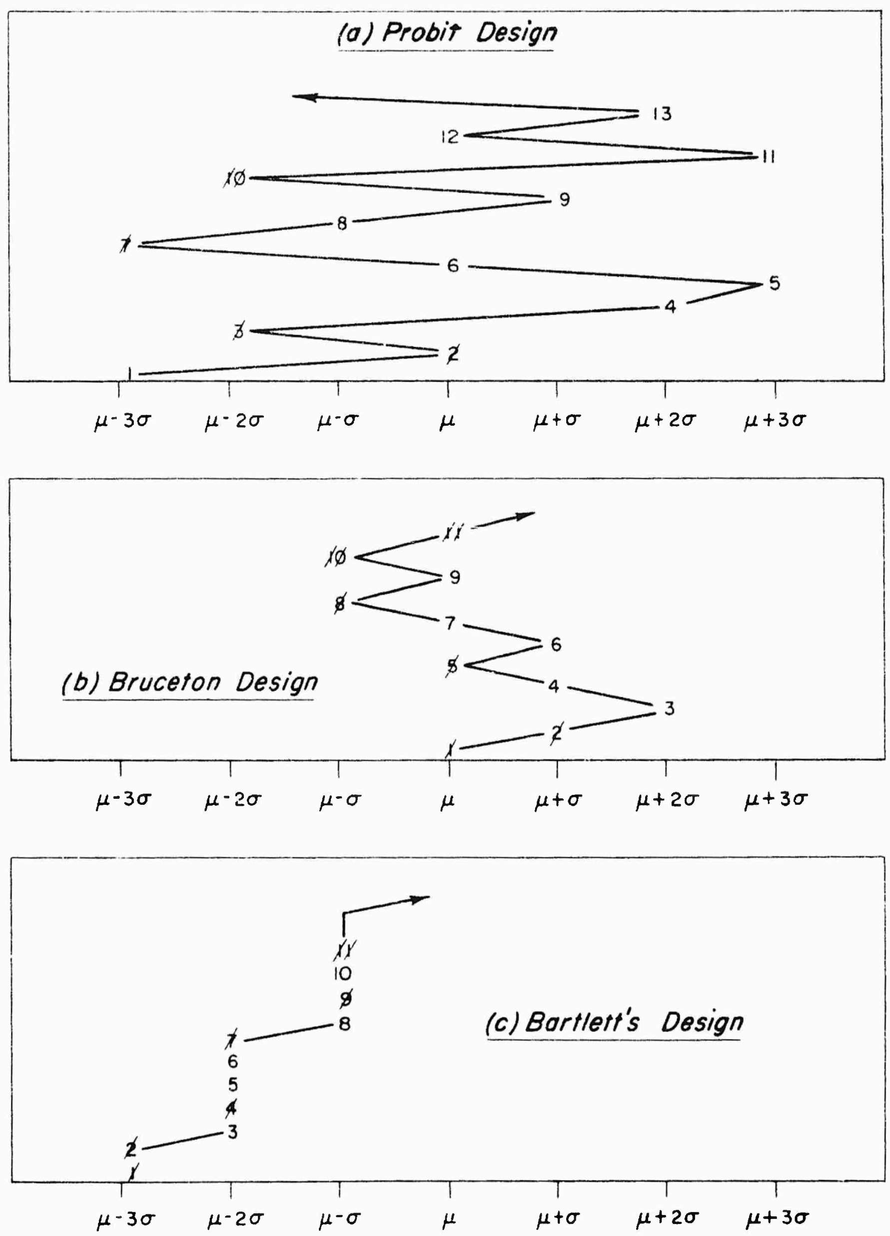
The probit design for the determination of parameters has been analyzed with regard to its efficiency. Under certain conditions it can be made very efficient in determining the mean, but some previous

Table 4-1. FIRING DATA OBTAINED DURING PROBIT CONTROL TEST OF BARTLETT TEST

DATE	TEMP.	HUMIDITY	DET. TYPE	DELAY (μsec)	LOT NO.	INITIALS	TEST NO.	TYPE OF TEST	PAGE
7-16-54 7-23-54			T18E4 R-Rejects	None			51	Bartlett, C = 0.001µfd.	231, 238

DET. NO.	RESISTANCE (OHMS)	FUNCT. TIME (μsec)	74.99	84.14	94.41	105.9	118.9	133.4	DET. NO.	RESISTANCE (OHMS)	FUNCT. TIME (μsec)	149.6	167.9	188.4	211.3	237.1	266.1
1			o						61			x					
2			o						62			o					
3			o						63			x					
4			o						64			x					
5			o						6'			o					
6			o									x					
7			o						6,			x					
8			o						68			o					
9			x						69			x					
10			o						70			x					
11				o					71				x				
12				o					72				x				
13				x					73				x				
14				o					74				x				
15				o					75				x				
16				o					76				x				
17				o					77				o				
18				o					78				o				
19				o					79				o				
20				o					80				x				
21					o				81					x			
22					o				82					x			
23					o				83					x			
24					o				84					x			
25					o				85					o			
26					o				86					x			
27					o				87					x			
28					o				88					o			
29					o				89					x			
30					o				90					x			
31						x			91						x		
32						x			92						x		
33						o			93						x		
34						o			94						x		
35						o			95						x		
36						o			96						x		
37						o			97						x		
38						o			98						x		
39						o			99						x		
40						o			100						x		
41							x		101							x	
42							o		102							x	
43							o		103							x	
44							x		104							x	
45							o		105							x	
46							o		106							x	
47							o		107							x	
48							o		108							x	
49							x		109							o	
50							o		110							x	
51								o	111								x
52								o	112								x
53								o	113								o
54								x	114								x
55								x	115								x
56								o	116								x
57								x	117								x
58								o	118								x
39								o	11-								x
60								x	120								x
n_x=			1	1	0	2	3	4	n_x=			7	7	8	10	9	9
n_o=			9	9	10	8	7	6	n_o=			3	3	2	0	1	1

EXPERIMENTAL DESIGNS FOR QUANTAL RESPONSE TESTS



NOTE: SLASHED NUMBERS (∅) INDICATE NEGATIVE RESPONSE

REPORT I-1804-1 FIGURE 4-2

DISTRIBUTION OF RESPONSES FOR THREE EXPERIMENTAL DESIGNS

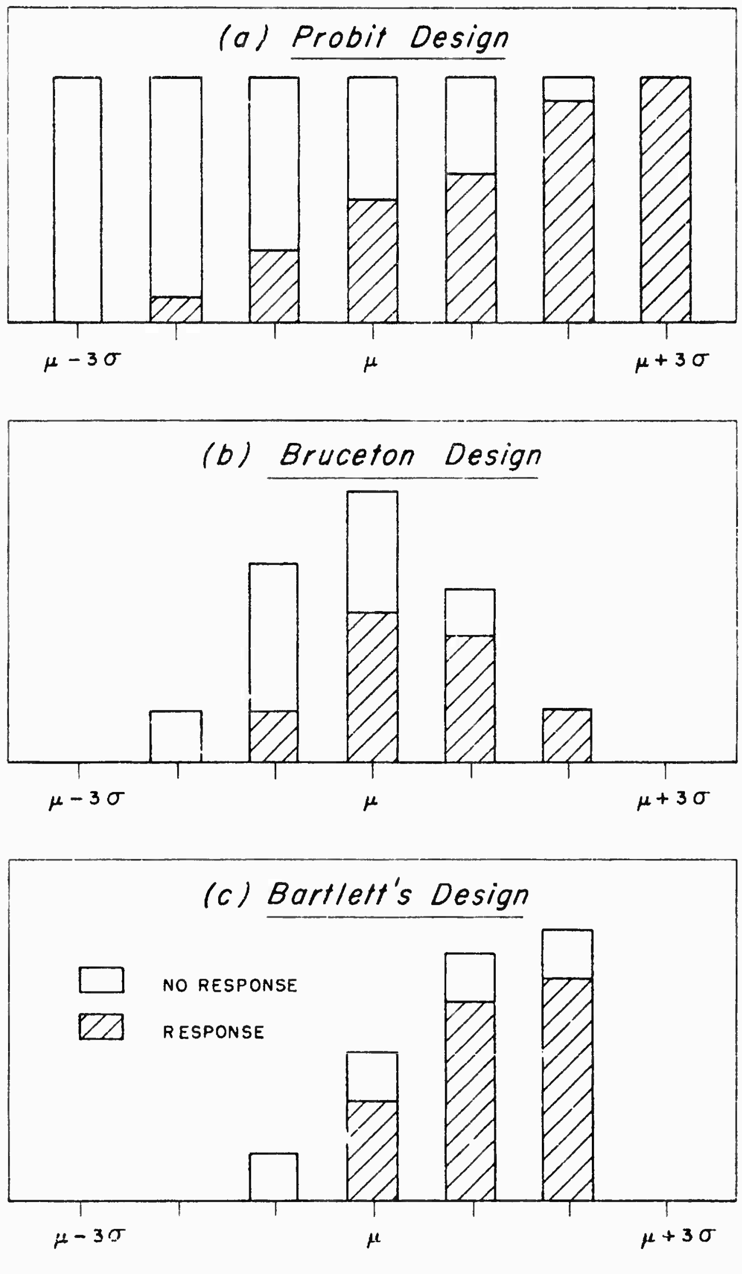


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FIGURE 4 - 3

knowledge about the distribution type is required. Generally speaking, however, it is not the most efficient design if all parameters are unknown. A probit design of two levels becomes most efficient in the determination of the standard deviation, however, if the mean is known. Since the latter event is rather improbable, the search for more efficient experimental designs has led statisticians to the following "up-and down" method.

4.3.2 The Bruceton Design

This design incorporates a sequential sampling technique. Figures 4-2 and 4-3 illustrate the organization and outcome of such an experiment. The basic idea is to perform tests at various stimulus levels. If any unit on a certain test level shows a negative response, the next unit is to be tested at the next higher level; otherwise it is to be tested at the next lower level. This design concentrates all the tests--and the information--at the mean and insures maximum efficiency for the estimate of this parameter.

The resultant sacrifice of efficiency in estimating the standard deviation can be made up by choosing a larger sample size. If two samples of identical size are tested with a probit and a Bruceton design, the latter is at least 30 per cent more efficient. It yields more efficient estimate of the mean and an equally good estimate of the standard deviation. Both efficiency estimates are asymptotic and the parameters should be obtained with maximum precision in the necessary calculations.

The Bruceton design is illustrated by data obtained with the T18E4 (R-Reject) detonators used extensively in the Bartlett design (Section 4.4.1). The charge capacitance was again 0.001 microfarads. Experimental data include functioning time for those units that did not fail, and resistance of the carbon bridge prior to firing. These data are reproduced in Table 4-2 as recorded during the test. Each "x" indicates that the particular unit functions, while the o indicates

Table 4-2. FIRING DATA OBTAINED DURING BRUCETON CONTROL TEST OF BARTLETT TEST

DATE	TEMP	HUMIDITY	DET. TYPE	DELAY (µsec)	LOT NO.	INITIALS	TEST NO.	TYPE OF TEST	PAGE
7-29-54	74°	54%	T18E4 R-Reject	none	-		61	Bartlett, C = 0.001µfd.	244

100.0	105.9	112.2	118.9	125.9	133.4	141.3	149.6	158.5	167.9	E	V	I	FUNCT. TIME (µsec)	RESISTANCE (OHMS)	DET. NO.
				o											1
					x										2
				o											3
					x										4
				x											5
			o												6
				x											7
			x												8
		o													9
			o												10
				o											11
					x										12
				o											13
					o										14
						o									15
							x								16
						o									17
							x								18
						o									19
							x								20
						o									21
							o								22
								o							23
									x						24
								o							25
									x						26
								x							27
							o								28
								o							29
									x						30
								x							31
							o								32
								o							33
									x						34
								x							35
							x								36
						o									37
							o								38
								x							39
							x								40
						o									41
							o								42
								o							43
									x						44
								x							45
							o								46
								x							47
							x								48
						o									49
							o								50
								o							51
									x						52
								x							53
							x								54
						x									55
					x										56
				o											57
					o										58
						x									59
					o										60
						o									61
							x								62
						x									63
					x										64
				x											65
			o												66
				o											67
					x										68
				x											69
			x												70
		o													71
			x												72
		x													73
	x														74
o															75
	o														76
		o													77
			o												78
				o											79
					o										80
						o									81
							o								82
								x							83
							x								84
						o									85
							o								86
								o							87
									x						88
								x							89
							x								90
						x									91
					o										92
						o									93
							o								94
								x							95
							o								96
								x							97
							x								98
						x									99
					x										100
															101
															102
															103
															104
															105
															106
															107
															108
															109
															110

	100.0	105.9	112.2	118.9	125.9	133.4	141.3	149.6	158.5	167.9	
n_x =	0	1	1	3	4	7	5	11	11	7	X
n_o =	1	1	3	4	7	5	11	11	7	0	O

malfunction. The table shows again that the Bruceton run gives a mean firing voltage of 136 volts. However, because of the small (logarithmic) increment in voltage, the test has several long runs. In such a case one frequently wonders whether the test has run into experimental difficulties such as instrumentational variations, non-randomness of the units etc., or whether such a run is "real" and can be statistically expected. The answer to this question requires statistical developments which exceed the scope of this report. However, we wish to point out that there is a likelihood that during this test instrumentational difficulties were encountered beginning with unit 21 and ending with unit 52; the same trouble seems to recur with detonator number 75. Our reason for such a statement is the observation that the detonator tends to have at first a lower mean firing level than from unit 21 on. It stabilizes at a relatively high firing level, then returns to the lower level, only to wander off to the higher level. In addition, it is known that the firing switch originally used in the Filits test unit acted up occasionally and might easily have caused the observed wandering in the Bruceton pattern. The analysis of these data is described in Section 4.4.3.

One who has had experience with the Bruceton method knows, of course, how to use it effectively to control quantal response experimental conditions. The shift in mean firing levels, the sudden appearance of long runs, alternate fires and misfires, etc. are all indications of changes in experimentation which bear watching and correcting. The great advantage of the Bruceton type experiment--its sensitivity to such experimental changes--over the probit type is that it supplies a steady source of information to the experimenter.

4.3.3 Bartlett's Design

A maximum amount of information about the tail-end or end point of the sigmoid distribution is obtained by Bartlett. His design requires that tests are first performed at levels near or below the mean response level. The tests are continued until exactly two failures have occurred. Then, testing continues at the next higher level, chosen at a suitable distance from the first level. Again, tests are made until exactly two failures have occurred.

This process is continued until the desired precision has been obtained. The levels are no longer chosen at random; they are assigned a definite sequence. Under no circumstances are the test levels ever lowered. The outcome of a test may or may not influence the level of the next test. Figures 4-2 and 4-3 give a graphical picture of this sequential test procedure. The last test on each level must be a negative response. The gain in efficiency of this method is impressive but costly. The number of units required at the higher test levels is almost prohibitive. At the 0.999 probability level thousands of units may be fired before there are two failures. It is obvious that small sample techniques are no longer adequate. However, this method is the most efficient known for getting narrow confidence bands at the high tolerance levels. No efficient design for small samples can yield high tolerance levels with great precision; only large samples based on a Bartlett-type design will give the required information on the model parameters.

The Bartlett type design can extend the gathered information to both tail-ends by carrying the tests successively to higher and lower levels. An example of such an experiment is given in Table 4-3 for the same T18E4 (R-reject) detonators that were used to illustrate the preceding designs. In this table we give, for the sake of brevity, only a summary of the test data. The first and the last level are both incomplete, as the supply of detonators became exhausted. A rough graphical evaluation of the data in Figure 4-4 shows the firing probabilities of this type detonator over the range tested. A more detailed evaluation of these data will be given in the appropriate subsection (4.4.4).

The detonators used in this test had been labelled resistance-rejected by the supplier because their resistances as measured by the

Table 4-3. SUMMARY OF FIRING DATA OBTAINED IN BARTLETT TEST T18E4 (R-REJECT), C = 0.001μfd

Log V	n_o	n_x	Log V	n_o	n_x
1.675	112	0	2.375	2	26
1.7	632	2	2.4	3	78
1.725	424	2	2.425	3	14
1.75	38	2	2.45	2	19
1.775	4	2	2.475	2	30
1.8	43	2	2.5	2	67
1.825	40	2	2.525	2	23
1.85	22	2	2.55	2	74
1.875	11	3	2.575	2	41
1.9	8	2	2.6	2	166
1.925	11	4	2.625	2	87
1.95	4	2	2.65	2	129
1.975	12	3	2.675	2	329
2.0	12	2	2.7	2	73
2.025	11	5	2.725	2	100
2.05	19	11	2.75	2	202
2.075	13	12	2.775	2	43
2.1	26	20	2.8	2	83
2.125	13	15	2.825	2	175
2.15	27	22	2.85	2	175
2.175	16	20	2.875	2	177
2.2	10	25	2.9	2	129
2.225	5	16	2.925	2	90
2.25	2	15	2.95	0	194
2.275	4	19			
2.3	2	12	Total:	1574	2788
2.325	2	17			
2.35	2	13			

manufacturer, did not seem to meet specifications. Upon measuring these resistances with our more refined instrumentation, we found that a majority of these detonators were within the specification range from 1000 to 10,000 ohms. Therefore, it appeared immediately that this detonator lot could be used to advantage to perform a Bartlett type test. Furthermore, the Bartlett design is independent of the type of unit tested. For example, if one lot of detonators were tested by means of a Bartlett test, the test results could be compared with those obtained for another lot. Since we did not have access to the needed, large quantities of detonators prior to this test, we offer the results obtained mainly as an illustration of the potentiality of which the Bartlett test is capable. It is not implied that the results obtained are descriptive of any other detonator lot, but the techniques demonstrated are applicable to other detonator lots as they become available for testing.

4.4 Analysis of Sensitivity Data

The experimental designs described in the preceding subsections will now be analyzed to yield the desired parameters. Most of the models underlying the analyses assume "normality" of the data or their distribution, and it will be necessary to make a few remarks concerning such an assumption. As stated earlier, (Sec. 4.2), the curves obtained from testing detonators are of the ogive type. In many instances, we can make transformations along the axis of test levels, or stimuli, such that the resulting curves resemble a cumulative, normal distribution curve. However, there does not exist any a priori knowledge about the type of transformation to be chosen, nor have the experiments performed thus far given any indication as to the exact or suspected nature of the type of distribution function which represents the data best.

Detonator users are extremely anxious to obtain information about the tail-end of these distributions since this is a region to which acceptance sampling tests and specifications are applied. Terms such as the "all-fire level" and the "no-fire level" are interpreted to mean something like a level at which 99.9 % or 0.1 % of the detonators fired can be expected to function. The all-fire level is used to insure proper installation and use of the detonator. The no-fire level is used to establish safe handling levels or discriminatory functioning levels for several types of detonators. Yet these levels are the most difficult to establish experimentally, and it is certainly true that they can hardly be determined within narrow confidence bands unless large numbers are tested.

In summary, the designs used to test detonators can be evaluated only against one another, and not against standards of any sort. Regardless of the assumptions made, these analyses are rather complicated, despite the simplicity of the concept. Each design is thus augmented by a form of data analysis, in addition to which there exists a general method of analysis developed by Berkson. These types of data analysis will be illustrated in the following sections and use will be made of various sets of test data that were available at the time the original analyses were made. In most instances we can compare the results with those for the T18E4 (R-reject) carbon bridge detonator used in the preceding section to illustrate the various experimental designs.

4.4.1 Probit Analysis

The probit analysis is applicable to data obtained by any of the preceding experimental designs. However, it is mostly used in conjunction with data obtained from a probit type design during which predetermined numbers of detonators are fired at predetermined stimulus levels. The probit analysis assumes that the data are of the normally distributed type. The first step, therefore, involves a graphical verification of that assumption. The fraction of units that are defective at each stimulus level is plotted essentially as a straight line on probability paper. The line can be tested for curvature by a Chi-Square test, and Finney has shown how a simple transformation of certain curved lines can often be made to yield a straight line for the transformed variable.

The evaluation of the data by the probit method is cumbersome. It leads to a maximum-likelihood estimate of the desired parameters and to a Chi-Square measure for significance tests. The process is illustrated here by data from the T18E3 (AAP-50-2) carbon bridge detonator. The firing data are shown in Table 4-4 and the corresponding evaluation by the probit method in Table 4-5. The latter table presents only the final iteration. Five steps were necessary to obtain stabilization of the iterative process.

The formulas shown in Table 4-5 are taken from Finney's "Probit Analysis". They correspond in meaning to the formulas that we shall develop later. The following parameters are of especial interest here:

$\bar{x}$ = mean logarithmic (transformed) firing voltage

b = slope of fitted probit line

a = intercept of fitted probit line

X^2 = weighted fitting error, an estimate of the deviation from the straight line

f = number of degrees of freedom

P = probability that the data can be represented by a straight line

V_{50} = mean firing voltage = antilog $\bar{x}$

$V_{.001}$ = firing voltage estimate for the 0.1 % firing level (with 50 % confidence)

$V_{.999}$ = firing voltage estimate for the 99.9 % firing level (with 50 % confidence)

$s_{V_{50}}$ = estimate for the standard error of the mean firing voltage

For the detailed computational method employed in this version of the probit analysis, we refer the reader to Finney's book on the subject. A modification of this analysis is described in a later section (4.4.4) of this report. This modification of the probit analysis does not take into account test levels that result in all failures or all fires, but it avoids the cumbersome iterations of Finney's method.

Table 4-4. EXPERIMENTAL DATA FOR THE T18E3 (AAP-50-2) CARBON BRIDGE DETONATORS BRUCETON RUN

C = 0.0196 microfarads

Detonator Number	Observed Functioning Time, (Micro-Seconds *)	Bruceton Firing Levels, Volts 100	79.5	63.2	50.2	39.9	31.7	25.2	Bridge Resistance, K Ohms
282-3	-						o		1.0
42-4	48.625					x			2.5
295-5	-						o		2.0
211-1	-					o			3.0
44-2	52.000				x				3.0
9-3	49.375					x			2.0
116-4	-						o		4.0
214-5	-					o			4.0
43-1	48.250				x				2.0
121-2	-					o			3.0
105-3	52.875				x				2.5
77-4	49.125					x			2.0
100-5	-						o		3.0
252-1	48.375					x			4.0
5-2	54.375						x		2.0
279-3	-							o	1.0
242-4	-						o		3.2
183-5	60.875					x			4.0
107-1	-						o		3.0
189-2	61.750					x			5.0
36-3	-						o		4.0
234-4	-					o			4.1
236-5	-				o				2.0
80-1	44.125			x					3.0
32-2	-				o				7.0
146-3	46.625			x					1.0
197-4	69.375				x				3.0
231-5	48.125					x			1.0
109-1	-						o		7.0
219-2	-					o			3.0
141-3	49.625				x				2.0
115-4	-					o			4.0
83-5	47.625				x				2.0
180-1	-					o			4.0
153-2	-				o				3.0
134-3	74.000			x					3.1
31-4	59.250				x				1.1
6-5	65.000					x			1.0
127-1	59.125						x		3.0
289-2	-								6.0
		Total x's		3	7	8	2	0	
		Total o's		0	3	7	8	2	

* Functioning times include a built-in delay of 43.375 microseconds

Table 4-5. PROBIT ANALYSIS OF T18E3 (AAP-50-2)

DATA FROM TABLE 4-4.

i	V	$x = \text{Log}\,\frac{V}{V_o}$	n_x	n	$p = \frac{n_x}{n}$	Y	nw	y	nwx	nwy
0	25.2	.00000	0	2	.00000	3.379	.4624	2.889	.0000	1.336
1	31.7	.09966	2	10	.20000	4.169	4.9299	4.159	.4913	20.504
2	39.9	.19957	8	15	.53333	4.962	9.5360	5.084	1.9031	48.481
3	50.2	.29930	7	10	.70000	5.752	5.1639	5.505	1.5456	28.427
4	63.2	.39932	3	3	1.00000	6.546	.7640	7.052	.3051	5.388

$$\Sigma nw = 20.856 \qquad \Sigma nwx^2 = 1.013$$
$$\Sigma nwx = 4.245 \qquad \Sigma nwy^2 = 530.10$$
$$\Sigma nwy = 104.136 \qquad \Sigma nwxy = 22.379$$

$$\bar{x} = \Sigma nwx \,/\, \Sigma nw = .2035$$

$$\bar{y} = \Sigma nwy \,/\, \Sigma nw = 4.993$$

$$b = [\Sigma nwxy\, \Sigma nw - \Sigma nwx\, \Sigma nwy] \,/\, [\Sigma nwx^2\, \Sigma nw - (\Sigma nwx)^2] = 7.931$$

$$a = \bar{y} - b\,\bar{x} = 3.379$$

$$\chi^2 = \Sigma nw\,(Y - y)^2 = \Sigma nwy^2 - a\,\Sigma nwy - b\,\Sigma nwxy = .76$$

$$f = 3$$

$$P = .85$$

$$x_{50} = (5 - a) \,/\, b = .2044$$

$$V_{50} = V_o \text{ antilog } x\,50 = 40.35$$

$$s_{a'}^2 = 1 \,/\, \Sigma nw = .0479$$

$$s_b^2 = 1 \,/\, \Sigma nw\,(x - \bar{x})^2 = 6.705$$

$$s_{x_{50}}^2 = s_{a'}^2 + (x_{50} - \bar{x})^2\, s_b^2 \,/\, b^2 = .000762$$

$$s_{V_{50}}^2 = s^2\, x_{50}\,(V_{50} \text{ lognat } 10)^2 = 6.58$$

$$s_{V_{50}} = 2.57 \qquad s_{a'} = .219$$

$$Y_{.001} = 1.9098$$
$$x_{.001} = -.1852$$
$$V_{.001} = 16.45$$

$$Y_{.999} = 8.0902$$
$$x_{.999} = .5940$$
$$V_{.999} = 98.96$$

The form of the probit analysis demonstrated here, as well as that shown later, has the advantage that deviations from the normal distribution can be treated after they show up. A transformation of the variate, or the fitting of another function to replace the straight line, will usually suffice. Either method of probit analysis is applicable to the analysis of quantal response data, whether the number of observations on each test level varies or not. The method can also be used to check results obtained by other forms of analysis, notably the Bruceton analysis.

Estimates of the efficiency of the probit design and the associated probit analysis have been made. It has been shown that even the best estimates of the 0.999 probability levels are affected by large sampling errors, no matter which of the proposed designs is chosen. Confidence bands at high tolerance levels are always extremely wide and the exact determination of high level tolerance points for purposes of quality control is very difficult. Furthermore, these estimates are made under the assumption that the distribution underlying the data is known to be normal. If this assumption is wrong, non-parametric estimates must be made and these are affected by even greater errors.

4.4.2 Berkson's Logit Analysis

This method of analysis applies--like the original probit analysis of Finney--to all types of experimental design discussed above. It develops a minimum logit Chi-Square estimate for the desired parameters. The method of calculation is simple and straightforward. Tables are employed but once, and the cumbersome iterations of the probit analysis are avoided. For maximum precision this method of analysis is preferable, although the underlying distribution is not exactly normal and deviates especially in the tail end.

For the details of this analysis, we refer the reader again to the original publication. The details of the computation, including the use of the published tables, are illustrated in Table 4-6. Of interest

Table 4-6. BERKSON'S ANALYSIS FOR T18E3 (AAP-50-2) DATA FROM TABLE 4-5.

i	V	$x = \log \frac{V}{V_1}$	n_x	n	$p = n_x / n$	nw	$nw\ell$	nwx	ℓ	$\hat{\ell} = a + b\,x$	$(\ell-\hat{\ell})^2$
0	25.2		0	2							
1	31.7	.00000	2	10	.20000	1.600	-2.218	.00000	-1.38629	-1.15494	.05352
2	39.9	.09992	8	15	.53333	3.733	.498	.37300	.13353	- .06479	.03933
3	50.2	.19965	7	10	.70000	2.100	1.779	.41926	.84730	1.02328	.03097
4	63.2		3	3							

$\Sigma nw = 7.433$
$\Sigma nwx = .79226$
$\Sigma nw\ell = .059$
$\Sigma nwx^2 = .12098$
$\Sigma nw\ell x = .40494$

$\bar{x} = \Sigma nwx / \Sigma nw = .10659$
$\bar{\ell} = \Sigma nw\ell / \Sigma nw = .00794$

$\Sigma nw\,(x-\bar{x})^2 = \Sigma nwx^2 - (\Sigma nwx)^2 / \Sigma nw = .03654$
$\Sigma nw(x-\bar{x})\,(\ell-\bar{\ell}) = \Sigma nw\ell x - \Sigma nw\ell\,\Sigma nwx/\Sigma nw = .39866$

$b = \Sigma nw\,(\ell-\bar{\ell})\,(x-\bar{x}) / \Sigma nw\,(x-\bar{x})^2 = 10.9102$
$a = [\Sigma nw\ell - b\,\Sigma nwx\,] / \Sigma nw = -1.15494$

$x_{50} = -a/b = .10585$
$V_{50} = V_1 \text{ antilog } x_{50} = 40.449$
$E_{50} = 5\,C\,V_{50}^2 = 160.34$

$\chi^2 = \Sigma nw\,(\ell-\hat{\ell})^2 = .2975$
$df = 5 - 2 - 2 = 1$
$p = .59$

$\ell_{.001} = -6.90675$
$x_{.001} = -.52719$
$V_{.001} = 9.42$

$\ell_{.999} = +6.90675$
$x_{.999} = .73891$
$V_{.999} = 173.8$

$s_{a'}^2 = 1/\Sigma nw = .13453$
$s_b^2 = 1/\Sigma nw\,(x-\bar{x})^2 = 27.3673$

$s_{x_{50}}^2 = [\,s_{a'}^2 + (x_{50} - \bar{x})^2\, s_b^2\,] / b^2 = .00113$
$s_{V_{50}}^2 = s_{x_{50}}^2\,(V_{50} \text{ lognat } 10)^2 = 9.8023$

$s_a' = .3668 \quad s_b = 5.231 \quad s_{x_{50}} = .0106 \quad s_{V_{50}} = 3.131$

are parameters similar to those discussed in the immediately preceding section of this report. A comparison between the results obtained will be made shortly.

4.4.3 Bruceton Analysis

The Bruceton Method of computation is based upon asymptotic formulas for maximum likelihood estimates. The underlying assumptions are that the samples are large and that the random variable has a normal distribution. A theory based on the use of small samples has been developed recently and yields similar formulas. The confidence bands for the extreme probability levels are very wide in either case. For the details of the developments, the reader is again referred to the pertinent original papers and publications.

A standard computational form was developed at the Franklin Institute Laboratories for the purpose of analyzing experimental data from detonator firings. This form is exhibited in Table 4-7 and contains data for the same T18E3 (AAP-50-2) carbon bridge detonator examined in earlier sections of this report. A summary of the analytical results obtained by various methods is presented in Table 4-9 for purposes of discussion. The computational form takes care of all cases where

$$n_o = n_x \text{ or } n_o \neq n_x .$$

Table 4-12 exhibits the former case.

4.4.4 Modified Probit and Logit Analyses

A method of simplifying the analysis of quantal response data was developed some time ago at The Franklin Institute Laboratories. Its details are given in Appendix B. By this method maximum likelihood estimates are derived for quantal response models of two forms:

$$\log [P / (1 - P)] = a_1 + b_1 \log V \tag{15}$$

and

$$u(P) = a_2 + b_2 \log V \tag{16}$$

Table 4-7. ANALYSES OF DETONATOR DATA BY BRUCETON METHOD

Type T18E3	**Manufacturer** Atlas	**Lot Number** AAP-50-2
Test Number	**Data Book Page** 51	**Detonator Nos.**
C = 0.0196 / μF	**Date Computed** 10-7-53	**Initials** CH

i	i^2	n_o	n_x	V = Volts
0	0	2	0	25.1
1	1	8	2	31.6
2	4	7	8	39.8
3	9	3	7	50.1
4	16	0	3	63.1
5	25			.
Totals : N_o = 20 N_x = 20				

Special Parameters

$c = (\log V)_{i=o} =$ 1.4

$d = (\log V)_{i+1} - (\log V)_i =$ 0.1

Test Conditions

Primary Statistics	**For "o's"**	**For "x's"**
$A = \Sigma i n$	31	51
$B = \Sigma i^2 n$	63	145
$m = c + d\ (A/N \pm 1/2)$ (+ for "o's", − for "x's")	1.605	1.605
$M = (NB - A^2)/N^2$	.7475	.7475
$\sigma = 1.62\ d\ (M + 0.029)$ for $M \geqslant 0.3$ only	.12579	.12579

Secondary Statistics		**V = Antilog m**	**W = 5 C V²**
$m = \dfrac{N_o m_o + N_x m_x}{N_o + N_x}$ =	1.605	40.27	158.9
$\sigma = \sqrt{\dfrac{N_o \sigma_o^2 + N_x \sigma_x^2}{N_o + N_x}}$ =	.12579		

Initials	**Page**

which represent the logit and the probit model, respectively. The parameters in each are obtained from maximum likelihood estimates (least squares) and can be found routinely, as shown in Table 4-8 which also exhibits the computational scheme that is used. Additional parameters, tolerance levels (such as the 50 % firing level and others) and confidence limits for these tolerance levels are obtained from the appropriate equations cited in Appendix B. The results of these calculations are then compared with the results of the preceding forms of analysis in Table 4-8.

The table indicates a number of interesting properties of the various forms of analysis. This comparison is only possible because the data chosen for analysis were of the Bruceton design type. Had we chosen data from a probit design, the Bruceton formulas would not have been applicable. Nevertheless, there are some facts of vital importance to the user of quantal response statistics which he should understand before he can proceed to make extensive experimental studies.

The analysis of experimental quantal response data requires some sort of a model. Whatever parameters are estimated from the data, are therefore estimated in conjunction with an implied model (thus far no satisfactory form of non-parametric quantal response statistics has been developed). If the model implies normality or symmetry of the underlying data, the estimates developed for certain high or low probability levels will exhibit a certain amount of symmetry about the estimated mean. If the model itself were skewed, then the estimates obtained from it will exhibit some asymmetry with respect to the mean.

With such thoughts in mind we shall now examine the data of Table 4-9. First, we notice that all methods of analysis agree pretty well on estimates of the mean firing level: The various estimates range from 40.35 to 40.73 volts. Second, the estimates for low and high probability firing levels obtained from the various forms of analysis are symmetric in a logarithmic sense; that is, the ratios of $V_{.001} / V_{.5}$ and $V_{.5} / V_{.999}$ are constant. The modified probit analysis agrees well with

Table 4-8. STANDARD COMPUTATIONAL FORM FOR MODIFIED LOGIT AND PROBIT ANALYSES

Logit and Probit Analyses	**Type** T18E3	**Manufacturer** Atlas	**Lot Number** AAP-50-2
Date Tested 10-7-53	**Test Number**	**Data Book Page** 51	**Date Computed** 10-7-53
Experimental Conditions	C = 0.0196 µf	**Remarks**	**Initials** CH

i	V	log V	n_o	n_x	$n=n_o+n_x$	$w=n_on_x/n$	w log V	$w \log(n_x/n_o)$	$w\,u(n_x/n)$	N_1	N_2	$(n_x-N_1)^2$	$(n_x-N_2)^2$
1	25.2	1.40140	2	0	2	0				.19182	.16172	.03679	.02615
2	31.7	1.50106	8	2	10	1.6	2.40170	-.96330	-1.34658	2.39486	2.32810	.15591	.10765
3	39.9	1.60097	7	8	15	3.73333	5.97695	.21650	.13391	7.25724	7.25925	.55169	.54871
4	50.2	1.70070	3	7	10	2.1	3.57147	.77276	1.10126	7.35726	7.28960	.12763	.08387
5	63.2	1.80072	0	3	3	0				2.67737	2.69973	.10409	.09016
6													
7													
8													
9													

k = 3	Σw = 7.43333	$\Sigma w \log V$ = 11.95012	$\Sigma w \log^2 V$ = 19.24801	$\Sigma w \log(n_x/n_o)$ = .02596	$\Sigma w\,u(n_x/n)$ = -.11141

$\Sigma w (\log V) \log(n_x/n_o)$ = .21487	$\Sigma w (\log V)\,u(n_x/n)$ = .06600	$\Delta = \Sigma w \Sigma w \log^2 V - (\Sigma w \log V)^2$ = .27144

$a_1 = (\Sigma w \log(n_x/n_o) \Sigma w \log^2 V - \Sigma w (\log V) \log(n_x/n_o) \Sigma w \log V)/\Delta$ = -7.61878	$N_1 = n/(1+\text{antilog}(-a_1-b_1\log V))$
$b_1 = (\Sigma w \Sigma w (\log V) \log(n_x/n_o) - \Sigma w \log V \Sigma w \log(n_x/n_o))/\Delta$ = 4.74128	$N_2 = n\,\phi(a_2 + b_2 \log V)$
$a_2 = (\Sigma w\,u(n_x/n) \Sigma w \log^2 V - \Sigma w (\log V)\,u(n_x/n) \Sigma w \log V)/\Delta$ = -10.80580	$V_1 = k\,\Sigma w (n_x-N_1)^2/((k-2)\Sigma w)$ = 1.040
$b_2 = (\Sigma w \Sigma w (\log V)\,u(n_x/n) - \Sigma w \log V \Sigma w\,u(n_x/n))/\Delta$ = 6.71221	$V_2 = k\,\Sigma w (n_x-N_2)^2/((k-2)\Sigma w)$ = .967

Table 4-9. COMPARISON OF PARAMETERS OBTAINED FROM VARIOUS FORMS OF PROBIT AND LOGIT ANALYSES FOR T18E3 (AAP-50-2) DETONATORS. [C = 0.0196μf]

	Probit Analysis	Logit Analysis	Modified Probit Analysis	Modified Logit Analysis	Bruceton Analysis
a	3.379	-1.1549	-10.8058	-7.6188	
b	7.931	10.9102	6.7122	4.7413	
m					2.2038
σ					.2516
$V_{.001}$	16.45	9.42	14.11	9.42	16.45
$V_{.5}$	40.35	40.45	40.73	40.45	40.27
$V_{.999}$	98.96	173.8	117.6	173.6	98.56

the original probit analysis. This agreement would be even better for probit experimental designs. The modified logit analysis agrees fully with the original analysis. This agreement is inherent in the use of computing machines instead of tables. Third, the most important point to be noticed is the disagreement between estimated extreme firing levels. This disagreement is basically inherent in the models. The reasons are thoroughly discussed in Appendix B; but even without reading this discussion we see that probit and logit models will lead their respective users to believe different things. For example, if it were known that a detonator obeys the probit law, the use of the probit analysis would yield correct tolerance levels. Use of the logit analysis would lead to incorrect and more widely spread tolerance levels. On the other hand, if a detonator were known to obey the logit model exactly, use of the logit equations would yield correct tolerance levels, while the use of probit equations would lead to erroneous and more narrowly spaced levels.

Therefore, it becomes increasingly important that we learn more about the actual distribution law which applies to detonators. However, this distribution cannot be ascertained from any small-scale probit or Bruceton test. Only the Bartlett test described previously, will enable us to separate the "right" from the "wrong" models with some ascertainable probability.

4.4.5 Analysis of Bartlett Design

The Bartlett design provides data such that the experimental evidence is weighted evenly on all test levels. By using additional levels, the "true" shape of the distribution curve can be determined to any desired degree of accuracy. However, it is likely that the detonator supply will be exhausted long before the nature of the distribution curve is firmly established. Even for somewhat limited experiments with a Bartlett design, various hypotheses can be compared and tested against one another. For example, one can determine whether the normal distribution is a valid hypothesis or must be rejected; whether the logit type of distribution is valid or must be rejected; or whether the probit type is significantly better or worse than the logit type.

Figure 4-4 shows the results of Table 4-3 in graphical form. We note that the observed experimental points show a different scatter for low voltages than for high voltages. Since all points have the same statistical weight, this change in scatter is significant. A curve drawn freehand through these points is not symmetric with respect to its center. This fact indicates a skewed distribution which differs from both the probit and the logit distributions.

If we make use of the available statistical tools, the following steps can be taken to extract information from our experimental data:

(a) We may apply a modified probit analysis to the aggregate of all data. Figure 4-4 tells us that such a fitting cannot be very good, but it establishes a yardstick with which we can measure other fittings.

(b) We may apply a modified logit analysis to the aggregate of all data. This procedure, too, cannot yield a very good fit because of the asymmetry exhibited in Figure 4-4. However, the resultant fit can be compared with that obtained for the probit analysis.

(c) We may apply modified logit and probit analyses to portions of the data, omitting the upper or the lower tail section. This procedure evaluates each of the two asymmetric branches separately. It yields better estimates of the parameters underlying the tail ends of the distribution.

In addition, we can compare results obtained from Bruceton and probit tests that were run concurrently with the Bartlett test. The comparison can be made for the basic parameters, or for the estimates of extreme firing levels. Disagreement between these estimates indicates that the assumed forms of the distribution function disagree and statistical tests

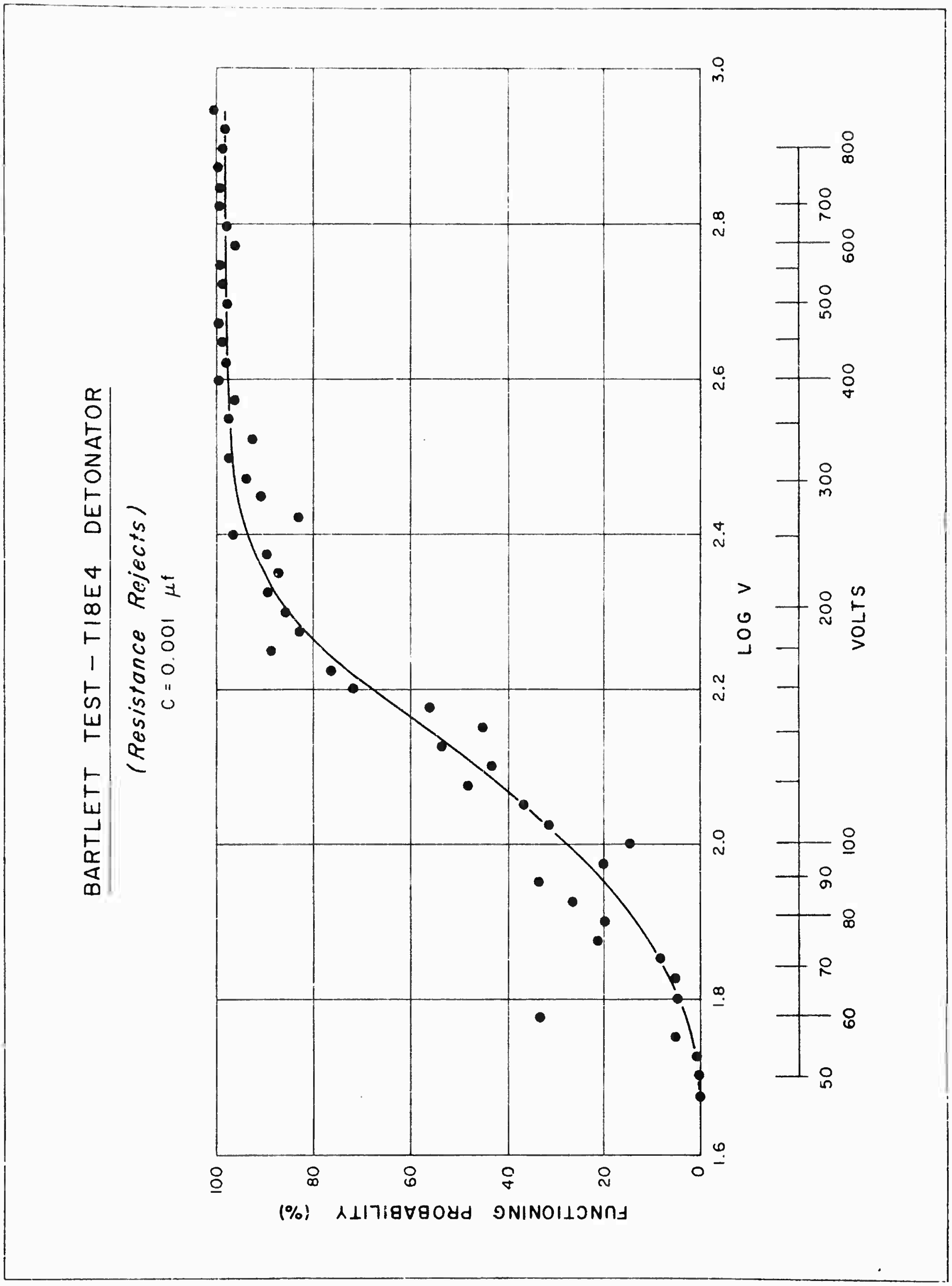

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FIGURE 4-4

must be applied in the search for a better fit to the data.

Table 4-10 shows details of the information obtained from the Bartlett test and control tests carried out in conjuction with it. The test procedure for this lot of T18E4 (R-reject) detonators involved firing at Bartlett levels. The mean was first established roughly by use of a Bruceton type test. In addition, a probit test was fired to obtain a check on the Bruceton tests. All these tests are summarized in Table 4-10. The table begins with the a column for the "pure" Bartlett test data which were obtained by firing at alternating high or low voltage levels until at least two fires and two misfires had occurred. The next column contains the probit test data used to check the sensitivity levels and calculated tolerance levels. The next two columns contain the two Bruceton tests, and a final column shows the grand total of all detonators fired at each level.

Table 4-11 exhibits the statistical parameters obtained from these data by various methods. Bruceton analyses were only computed for the two Bruceton type experimental designs. However, modified probit and logit analyses were computed for all designs and can now be compared with Bruceton analyses and with one another. The table confirms a number of facts known from statistical theory. We shall list them here separately for future reference:

(a) Bruceton designs are very sensitive to the choice of the correct "d". If the value of "d" is too small, the computed Bruceton-Sigma will be much smaller than the Sigma obtained from the probit analysis, although the former is an asymptotic estimate of the latter. The estimated tolerance levels for Bruceton designs with small d-values lie too close to the mean. (cf. $V_{.001}$, $V_{.999}$ in first and second Bruceton tests).

(b) If the firing data indicate a skewed distribution, estimates of the mean firing levels from Bruceton, probit, and logit analyses will not agree very well. If the firing data indicate

Table 4-10. SUMMARY OF BARTLETT TEST AND BRUCETON OR PROBIT CONTROL TEST DATA

T18E4 (R-Reject) CARBON BRIDGE DETONATORS

(C = 0.001 µf)

Firing Voltage		Bartlett Test				Probit Test				First Bruceton Test (d = .025; page 244)			Second Bruceton Test (D = .05; page 249)			Grand Total (As in Table 4-8 and Figure 4-12.			
V	log V	n_o	n_x	n	Data Pages	n_o	n_x	n	Data Pages	n_o	n_x	n	n_o	n_x	n	n_o	n_x	n	$P_x = \frac{n_x}{n}$ Percent
47.32	1.675	112	0	112	255,256											112	0	112	0
50.12	1.7	632	2	634	250-255											632	2	634	.32
53.09	1.725	424	2	426	204,229-233											424	2	426	.47
56.23	1.75	38	2	40	204											38	2	40	5
59.57	1.775	4	2	6	204											4	2	6	33.33
63.10	1.8	43	2	45	203-204											43	2	45	4.44
66.83	1.825	40	2	42	203											40	2	42	4.76
70.80	1.85	22	2	24	203											22	2	24	8.33
74.99	1.875	2	2	4	202	9	1	10	231							11	3	14	21.43
79.43	1.9	8	2	10	202											8	2	10	20
84.14	1.925	2	3	5	202	9	1	10	231							11	4	15	26.67
89.12	1.95	4	2	6	202											4	2	6	33.33
94.41	1.975	2	3	5	202	10	0	10	231							12	3	15	20
100.0	2.0	4	2	6	202					1	0	1	7	0	7	12	2	14	14.29
105.9	2.025	2	2	4	202	8	2	10	231	1	1	2				11	5	16	31.25
112.2	2.05	2	3	5	202					3	1	4	14	7	21	19	11	30	36.67
118.8	2.075	2	6	8	202	7	3	10	231	4	3	7				13	12	25	48
125.9	2.1	3	2	5	201					7	4	11	16	14	30	26	20	46	43.48
133.4	2.125	2	4	6	202	6	4	10	231	5	7	12				13	15	28	53.57
141.3	2.15	3	2	5	201					11	5	16	13	15	28	27	22	49	44.90
149.6	2.175	2	2	4	202	3	7	10	231	11	11	22				16	20	36	55.56
158.5	2.2	2	2	4	201					7	11	18	1	12	13	10	25	35	71.43
167.9	2.225	2	2	4	202	3	7	10	231	0	7	7				5	16	21	76.19
177.8	2.25	2	14	16	201								0	1	1	2	15	17	88.24
188.4	2.275	2	11	13	202	2	8	10	238							4	19	23	82.61
199.5	2.3	2	12	14	201											2	12	14	85.71
211.4	2.325	2	7	9	202	0	10	10	238							2	17	19	89.47
223.9	2.35	2	13	15	201											2	13	15	86.67
237.1	2.375	2	17	19	203	1	9	10	238							3	26	29	89.66
251.2	2.4	2	69	71	201-202	1	9	10	231							3	78	81	96.30
266.1	2.425	2	5	7	204	1	9	10	238							3	14	17	82.35
281.8	2.45	2	19	21	204											2	19	21	90.48
298.5	2.475	2	30	32	233-234											2	30	32	93.75
316.2	2.5	2	67	69	234											2	67	69	97.10
335.0	2.525	2	23	25	234											2	23	25	92
354.8	2.55	2	74	76	234-235											2	74	76	97.37
375.8	2.575	2	41	43	235-236											2	41	43	95.35
398.1	2.6	2	166	168	236-237											2	166	168	98.81
421.7	2.625	2	87	89	237-238											2	87	89	97.75
446.7	2.65	2	129	131	239-240											2	129	131	98.47
473.2	2.675	2	329	331	240-243											2	329	331	99.40
501.2	2.7	2	73	75	245											2	73	75	97.33
530.9	2.725	2	100	102	245-246											2	100	102	98.04

Table 4-10. SUMMARY OF BARTLETT TEST AND BRUCETON OR PROBIT CONTROL TEST DATA (CONTD.)

T18E4 (R-Reject) CARBON BRIDGE DETONATORS

(C = 0.001 μf)

Firing Voltage		Bartlett Test				Probit Test				First Bruceton Test (d = .025; page 244)			Second Bruceton Test (d = .05; page 249)			Grand Total (As in Table 4-8 and Figure 4-12)			
V	log V	n_o	n_x	n	Data Pages	n_o	n_x	n	Data Pages	n_o	n_x	n	n_o	n_x	n	n_o	n_x	n	$P_x = \frac{n_x}{n}$ Percent
562.3	2.75	2	202	204	246-248											2	202	204	99.02
595.7	2.775	2	43	45	257											2	43	45	95.56
631.0	2.8	2	83	85	257-258											2	83	85	97.65
668.4	2.825	2	175	177	258-260											2	175	177	98.87
707.9	2.85	2	175	177	260-261											2	175	177	98.87
749.9	2.875	2	177	179	261-263											2	177	179	98.88
794.3	2.9	2	129	131	263-264											2	129	131	98.47
841.4	2.925	2	90	92	264-265											2	90	92	97.83
891.3	2.95	0	194	194	265-267											0	194	194	1.
															TOTALS	1574	2788	4362	

Table 4-11. STATISTICAL PARAMETERS AND TOLERANCE LEVEL FOR VARIOUS PARTS OF THE BARTLETT TEST DATA

T18 E4 (R - Reject) Detonator

C = 0.001 μf

		Probit Test	First Bruceton Test (d = .0225)	Second Bruceton Test (d = .05)	Combined Bruceton Tests	Grand Total of All Data	Grand Total Less First Six Lines
a_1		-8.67557	-5.02653	-11.43613	-7.68451	-6.78955	-6.31796
b_1		4.04448	2.31997	5.40940	3.59990	3.14629	2.95230
σ^*_1		.17910	.31223	.13391	.20122	.23023	.24536
υ_1		.406	2.447	3.681	6.819	15.530	6.26119
$V_{.001}$	LOGIT ANALYSIS	25.32	7.476	36.28	20.02	16.02	13.30
$V_{.01}$	LOGIT ANALYSIS	44.84	20.25	55.62	38.04	33.39	29.11
$V_{.05}$	LOGIT ANALYSIS	67.43	41.25	75.46	60.18	56.43	50.92
$V_{.5}$	LOGIT ANALYSIS	139.6	146.8	130.0	136.3	143.9	138.0
$V_{.95}$		289.2	522.2	224.1	308.9	366.7	374.2
$V_{.99}$		435.0	1064.	304.1	488.7	619.8	654.6
$V_{.999}$		770.3	2881.	466.3	928.7	1292.	1432.
Key in Figure 4-5		P_ℓ	$B_{1\ell}$	$B_{2\ell}$	B_ℓ	T_ℓ	$T_{-\ell}$
a_2		-11.80176	-7.16858	-15.81367	-10.93359	-8.00051	-7.34005
b_2	PROBIT ANALYSIS	5.50195	3.31816	7.47697	5.12128	3.75051	3.47938
σ^*_2	PROBIT ANALYSIS	.18175	.30137	.13374	.19526	.26663	.28740
υ_2	PROBIT ANALYSIS	.505	2.454	3.192	6.759	28.617	8.36884
$V_{.001}$	PROBIT ANALYSIS	38.31	17.18	50.31	34.00	20.38	16.65
$V_{.01}$	PROBIT ANALYSIS	52.73	29.18	63.64	47.92	32.56	27.59

Table 4-11 - (Continued)

		Probit Test	First Bruceton Test (d = .0225)	Second Bruceton Test (d = .05)	Combined Bruceton Test	Grand Total of All Data	Grand Total Less First Six Lines
PROBIT ANALYSIS	$V_{.05}$	70.15	46.84	78.52	65.12	49.50	43.33
	$V_{.5}$	139.6	146.7	130.3	136.4	135.9	128.7
	$V_{.95}$	278.0	459.4	216.3	285.8	373.1	382.3
	$V_{.99}$	369.8	737.4	266.8	388.4	567.0	600.3
	$V_{.999}$	508.9	1252.	337.5	547.4	906.0	994.8
	Key in Figure 4-5	P_p	B_{1p}	B_{2p}	B_p	T_p	T_{-p}
	m		2.14800	2.11150			
	σ		.16332	.09125			
BRUCETON ANALYSIS	$V_{.001}$		43.98	67.53			
	$V_{.01}$		58.61	79.28			
	$V_{.05}$		75.74	91.49			
	$V_{.5}$		140.6	129.3			
	$V_{.95}$		261.0	182.6			
	$V_{.99}$		337.3	210.8			
	$V_{.999}$		449.5	247.4			
	Key in Figure 4-5		B_{1b}	B_{2b}			

a symmetric distribution, possibly, under proper transformation, estimates of the mean firing levels will agree well. Therefore, extreme tolerance level estimates will disagree or agree (as the case may be) if various types of analyses are used on the same data, depending on the type of distribution.

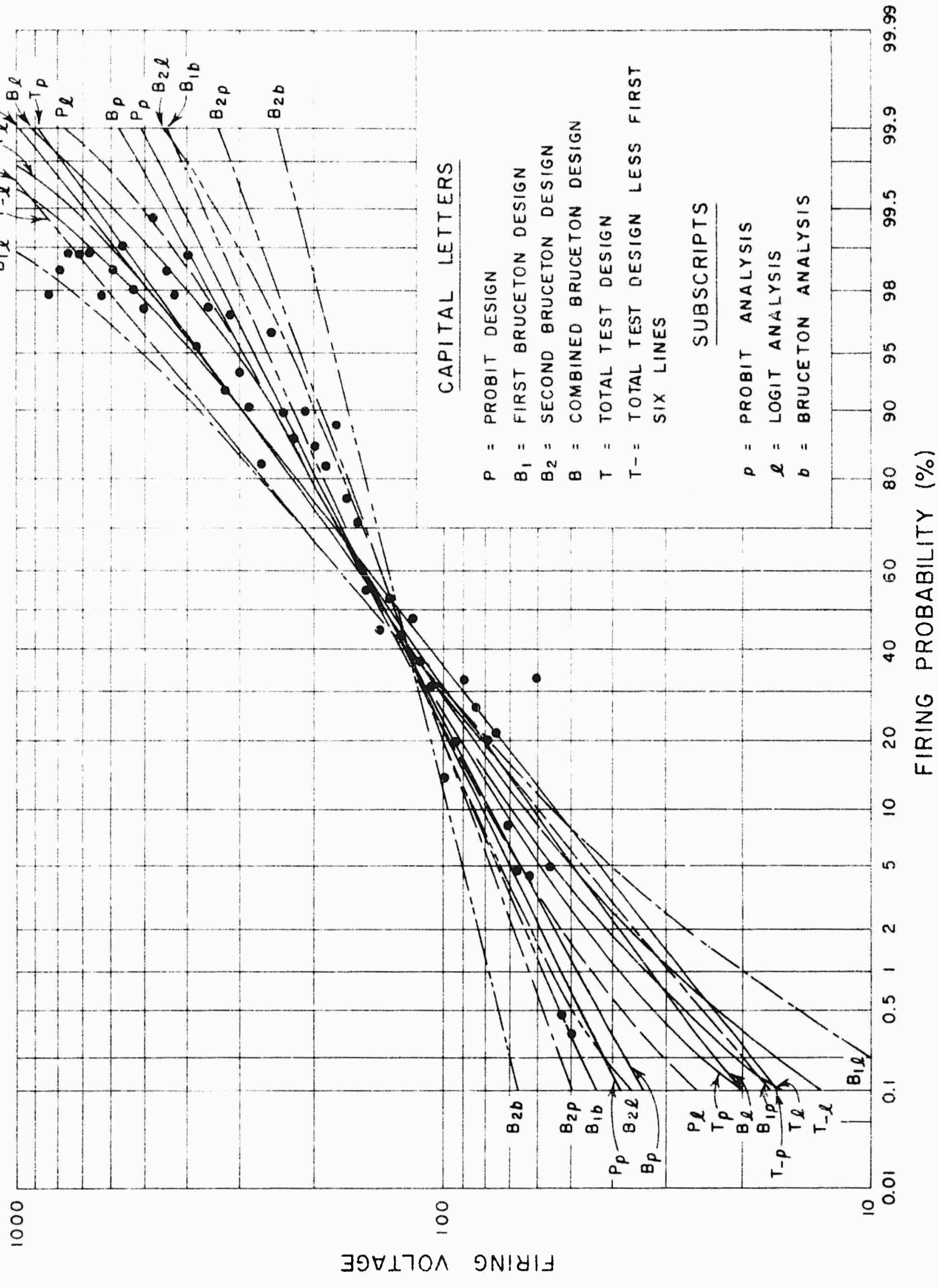
(c) The variance ratio, ν_1 / ν_2, indicates that any combination of Bruceton or probit designs and modified logit or probit analyses does not yield a fit that is significantly different. Estimates of tolerance levels made from either form of analysis cannot be statistically distinguished from each other, even though they differ in magnitude.

(d) The variance ratio ν_1 / ν_2 for the total data (also for the data reduced by the first six lines) indicates that the logit type of analysis produces a significantly better fit to the data than the probit type. Estimates of high or low tolerance levels for this type detonator, therefore, are more reliable if based on the logit type analysis.

(e) The variance reduction incurred by omitting the first six lines from the grand total of all data confirms the skewness of the observed distribution. Estimates of high or low tolerance levels, therefore, are better obtained from reduced sets, if observations are made at both extremes of firing probability.

Figure 4-5 depicts on probability paper the results of the various analyses and the observed experimental points. The choice of this type scale obviously yields straight lines for probit and Bruceton type analyses, while logit type analyses yield slightly curved lines. This particular behavior of the fittings is studied in greater detail in Appendix B. Figure 4-5 shows how the significance statement made under (d) above is borne out by a graphical analysis. The experimental points indicate a definite bend, especially in the region of high functioning levels. The curvature introduced by the logit type analysis is not

COMPARISON OF BARTLETT TEST DATA WITH VARIOUS CONTROL TESTS AND ANALYSES



REPORT I-1804-1 FIGURE 4-5

sufficient to follow the curvature exhibited by the data points. Therefore, we shall want to investigate more closely the trend indicated by these experimental points.

Scrutiny of the data reveals some startling results. We are sure that for this lot of detonators neither probit nor logit analysis applies exactly, although the latter produces a significantly better fit. This is because the distribution is definitely not symmetric on a log V scale. It breaks off sharply at the lower end (cf. Fig. 4-4), but rises slowly near the high end of the log V scale. This pattern of the experimental points is not hard to understand. Assume, for the sake of an argument, that this lot contains a relatively small percentage of "perfect" duds.* What would happen to our curve? At the low end, it seems, all detonators reach a point below which they will not fire . The duds won't trouble us much there, since both duds and good detonators just don't fire below, say, 46 volts. The performance is different at the high end of the curve. There, the good detonators should all fire; but a persistent (though small) percentage of duds keeps on cropping up and causes the functioning probability to stay below unity. This describes exactly the performance of the units tested. It need not necessarily be true for other detonators or other manufacturers; but we are on safe ground when we say that the presence of a small percentage of duds will skew the distribution curve in such a fashion as this.

To illustrate this point, we have constructed Figure 4-6. The same experimental points, shown in Figure 4-5 are now interpreted by a series of asymptotes and graphically-estimated probability lines. If this is the kind of model which describes the response of these detonators, a considerable amount of theoretical work will be necessary before a

---

* A dud may be regarded as an assembly, externally indistinguishable from a detonator, but incapable of detonating because of some flaw in its interior construction. Such a flaw might be the lack of a wire or a carbon bridge, the lack of charge, a wet charge, a charge consisting of inert material instead of an explosive, etc.

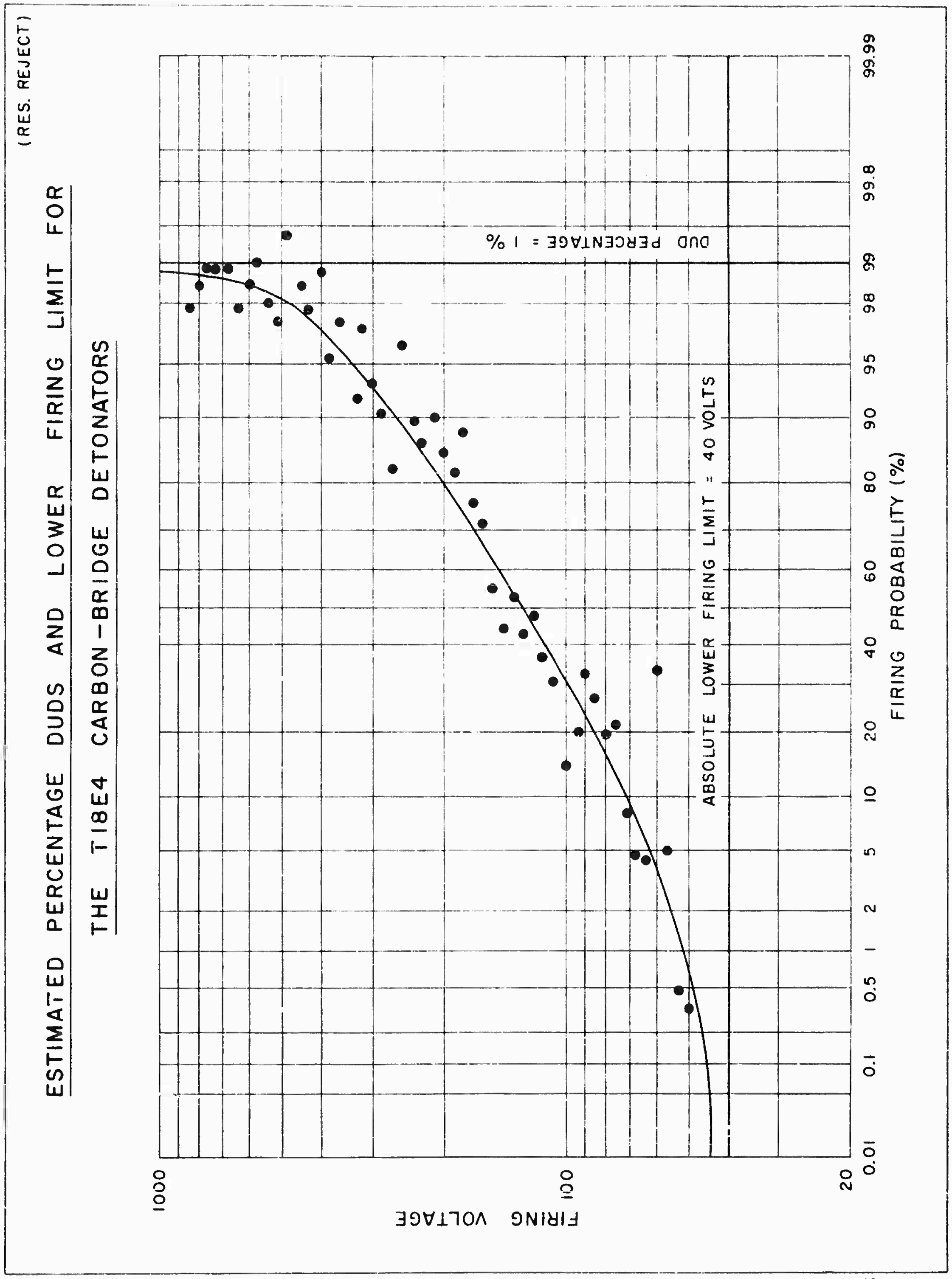


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FIGURE 4-6

satisfactory interpretation of experimentally obtained firing data is found. Several attempts at dud analysis have been made, but none have yet given satisfactory results. A detailed study of this problem must be made in the future.

4.5 Graphical Evaluation of Sensitivity Data

Sensitivity data are quantal response data and were discussed in the preceding sections of this report. In these discussions we have restricted the evaluation of pertinent statistical parameters to the case where the input conditions have only one variable. For example, the quantal response curves were obtained for the T18E4 carbon bridge detonator by firing from the fixed capacitance, C = 0.001 microfarads, at various voltages. These voltages were varied according to some experimental design which makes it possible for us ultimately to estimate the voltage at which the functioning probability of the detonator has some given value. For example, Figure 4-6 shows that the T18E4 (R-Reject) detonator has a functioning probability of 98% at a firing voltage of 500 volts and a capacitance of C = 0.001 microfarads. It is much easier, however, to find from Table 4-11 (First Bruceton test and Bruceton analysis) that for this detonator we have the following combination of statistical parameters: $(C,m,\sigma) = (0.001, 2.148, 0.16332)$. It follows that $V_{.5} = 140.6$ volts, $V_{.999} = 449.5$ volts, etc. In other words, if we wish to characterize a detonator by varying more than one parameter, we can state the results in tabular form and proceed with the analysis of these data on some convenient basis.

Table 4-12 contains such data, obtained in Test 44 for the T18E4 (R < Specs; Special) lot of carbon bridge detonators. The tests were of the Bruceton design, and were analyzed according to the scheme exhibited in Table 4-7. Table 4-12 indicates the well-known dependence of the mean firing voltage $V_{.5}$ on firing capacitance. The greater the capacitance, the smaller the mean firing voltage $V_{.5}$ or its logarithm, m. There is some indication that the so-called sigma, obtained from the

Bruceton analysis, is also dependent on the firing capacitance. We have tested several other detonators for this dependence, but have not yet been able to state conclusively what the dependence is. Variance ratio tests used on the sigmas of Table 4-12 do not result in statements concerning statistically significant sigmas for the various capacitances. Therefore, we cannot reject the hypothesis that the observed variation in sigmas is only due to sampling variations, and shall proceed to estimate an average, or over-all sigma σ from

$$\overline{\sigma} = \sqrt{\Sigma N_C \, \sigma^2_C \, / \Sigma N_C} \qquad (17)$$

where N_C represents the number of detonators used at each C-level, and σ_C is the sigma-value obtained at a particular C-level. This average is only asymptotically correct, but we feel justified in using this short-cut formula. Equation (17) yields a value of $\overline{\sigma} = 0.0896$ which is shown at the bottom of Table 4-12. We can perhaps devise a better formula than equation (17) by referring to Hald's "Statistical Theory with Engineering Applications."

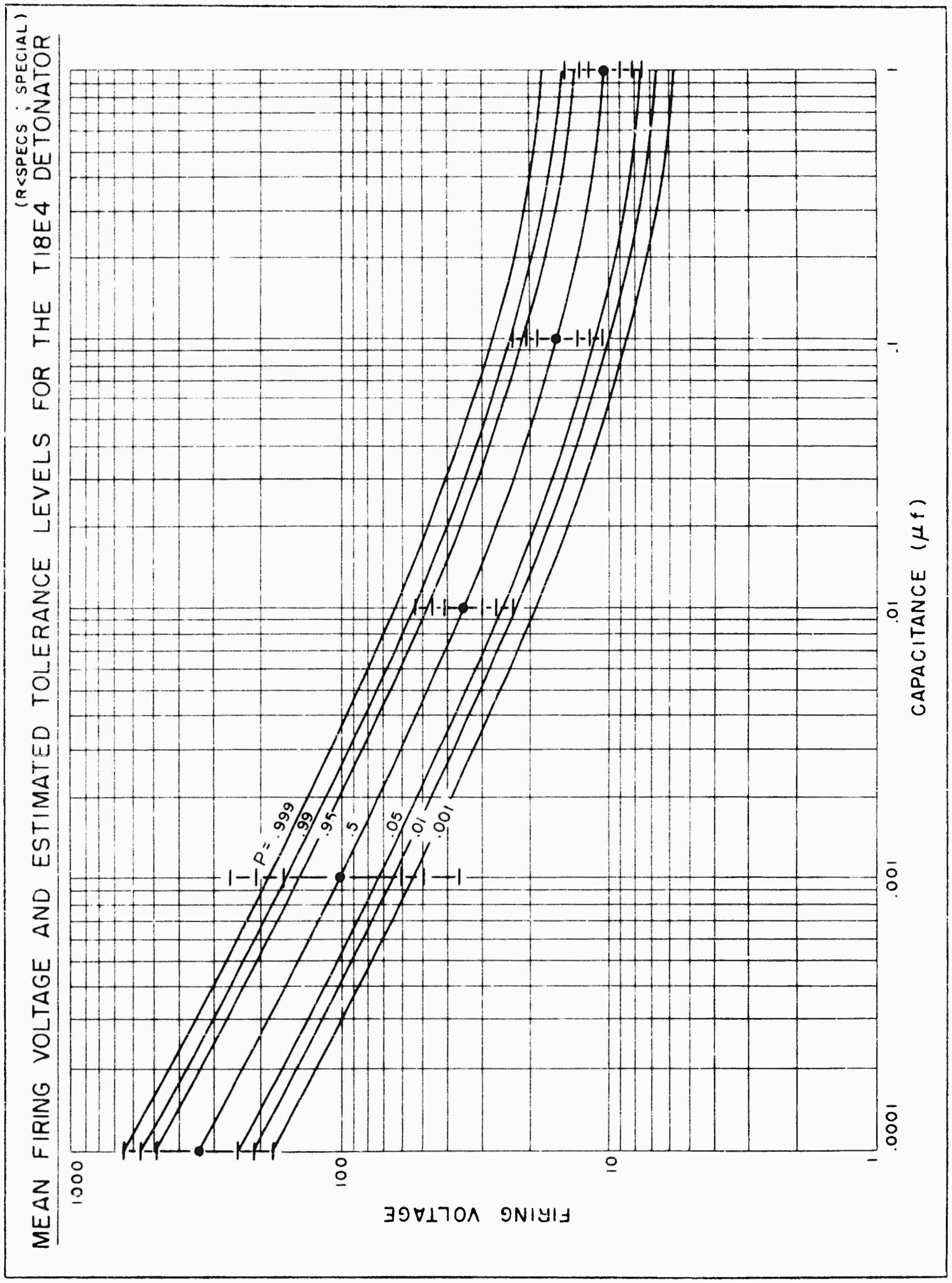
As in the case of the mean functioning times, we can now prepare a plot of the mean firing voltages $V_{.5}$ on logarithmic graph paper. The vertical and horizontal scales are respectively log V and log C. Such a plot is shown in Figure 4-7 for the T18E4 (R < Specs; Special) data given in Table 4-12. From the plotted points of $V_{.5}$ we obtain readily the entire curve between C = 0.0001 and C = 1.0 microfarads, labelled P = 0.5. The curve is smooth and shows no deviation from the plotted points.

Next, we want to plot the tolerance levels, labelled P = 0.999, 0.99, etc. For comparison, we may want to plot both the individual (local) tolerance levels as well as the smoothed levels obtained from equation (17). In either case, we have to translate the logarithmic values of firing voltage, m, and σ into antilogarithmic voltages. If we use again the multiples of σ shown on page 20, namely $t_p = 1.645$,

Table 4-12. SUMMARY OF RESULTS OBTAINED FROM BRUCETON TESTS ON T18E4 (R < SPECS; SPECIAL) CARBON BRIDGE DETONATORS

Firing Capacitance C (μf)	Number of Detonators Used in test	$m = \log V_{.5}$	$V_{.5}$	σ
1.0	40	0.95062	8.925	0.0635
0.1	40	1.17191	14.86	0.0592
0.01	40	1.51875	33.02	0.0627
0.001	40	1.98375	96.33	0.1429
0.0001	40	2.5369	344.3	0.0908

$\bar{\sigma} =$ 0.0896



MEAN FIRING VOLTAGE AND ESTIMATED TOLERANCE LEVELS FOR THE T18E4 DETONATOR (R<SPECS ; SPECIAL)

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FIGURE 4-7

2.327, 3.090, we have

$$V_p = \text{Antilog}\,(m + t_p\,\bar{\sigma}) \tag{18}$$

As can be expected, the locally obtained levels deviate from the average tolerance levels in both directions, but the deviations are not large enough to warrant the assumption of a sigma that is variable with firing capacitance. However, such variations have been observed and should be investigated carefully in either graphical or analytical evaluation of quantal response firing data.

These simple steps complete the graphical sensitivity analysis. Similar curves have been prepared for all detonators evaluated at the Franklin Institute and are now familiar to most users. The essential feature of these diagrams is a characteristic family of curves with like firing probability which can be obtained for each initiator. These curves can be written in a general form by stating that the firing voltage V is a function of the firing capacitance C and the desired firing probability P: V = V(C, P). If stated thus, the relation assumes the explicit form of an equation in three variables which represents a surface in space. The cross-sections of this surface are then plotted on a suitable scale, such as logarithmic scales for V and C in case of the detonators studied here. In the next section we shall make use of the knowledge that this general relationship exists. We shall try to elicit it from the data by analytical, rather than graphical methods.

4.6 Analytical Evaluation of Sensitivity Data

In Section 4.5 we have described in detail methods whereby a graphical analysis can be made of the quantal response firing data of detonators. The essense of this analysis is the determination of a function—in graphical form, of course—which relates firing voltage, firing capacitance, and firing probability: V = V(C, P). This situation is different from that encountered in the study of functioning time data since we now deal with an attribute and not with a variable. Nevertheless, there are some points of similarity which will help us understand the following analysis more readily.

First, we shall again deal with a logarithmic model instead of the raw data model: log V = log V(log C, P). One glance at Figure 4-7 tells us that the shape of these curves is relatively simple. Of course, we have had the advantage of studying scores of detonator graphs in the past and they all indicate the same thing: the resultant P-curves tend to rise for decreasing log C-values, and flatten out for increasing log-C values. We shall try to interpret this information later (cf. Resumé, Sect. 4.7). To introduce the model here, it will be sufficient to state that the above functional relationship seems to be analytic and therefore, can be developed into a complete Taylor series. In this development, there must appear some function of the firing probability; this can be the so-called normal function, the logit function, or any other function that some future investigation may reveal to be superior to these two. For details with regard to these two functions, we refer the reader to what has been said in Sections 4.4, 4.5 and Appendix B.

If we denote this probability function in general by F(P), we find that F(P) can generally be written as F(P) = a + b log V, for constant firing capacity. The complete Taylor expansion for F(P) requires, therefore, only additional terms in log C:

$$F(P) = a + b \log V + c_1 \log C + c_2 \log^2 C + c_3 \log^3 C + \ldots \qquad (19)$$

If F(P) is highly skewed with respect to log V, additional terms involving $\log^2 V$, log V log C, etc., may have to be introduced. However, the present analysis was restricted to the study of symmetric probability functions, specifically to the logit and probit models for F(P).

The procedure for finding the parameters (a, b, c_i) is briefly the following: The function (19) is fitted to the given data by taking successively more terms. First, a simple model

$$F_1(P) = a_1 + b_1 \log V + c_{11} \log C \tag{20.1}$$

is fitted to the observed quantal response data by least-squares technique. The fitting of course, leaves a certain amount of unexplained deviation between the model and the actual data. This amount is computed in the form of a variance. Since the model (20.1) implies a straight-line relationship on logarithmic paper, we realize at once that it will not fit the data very well, but it may be used as a null-hypothesis to test other models. Next, we may try a more refined model:

$$F_2(P) = a_2 + b_2 \log V + c_{21} \log C + c_{22} \log^2 C. \tag{20.2}$$

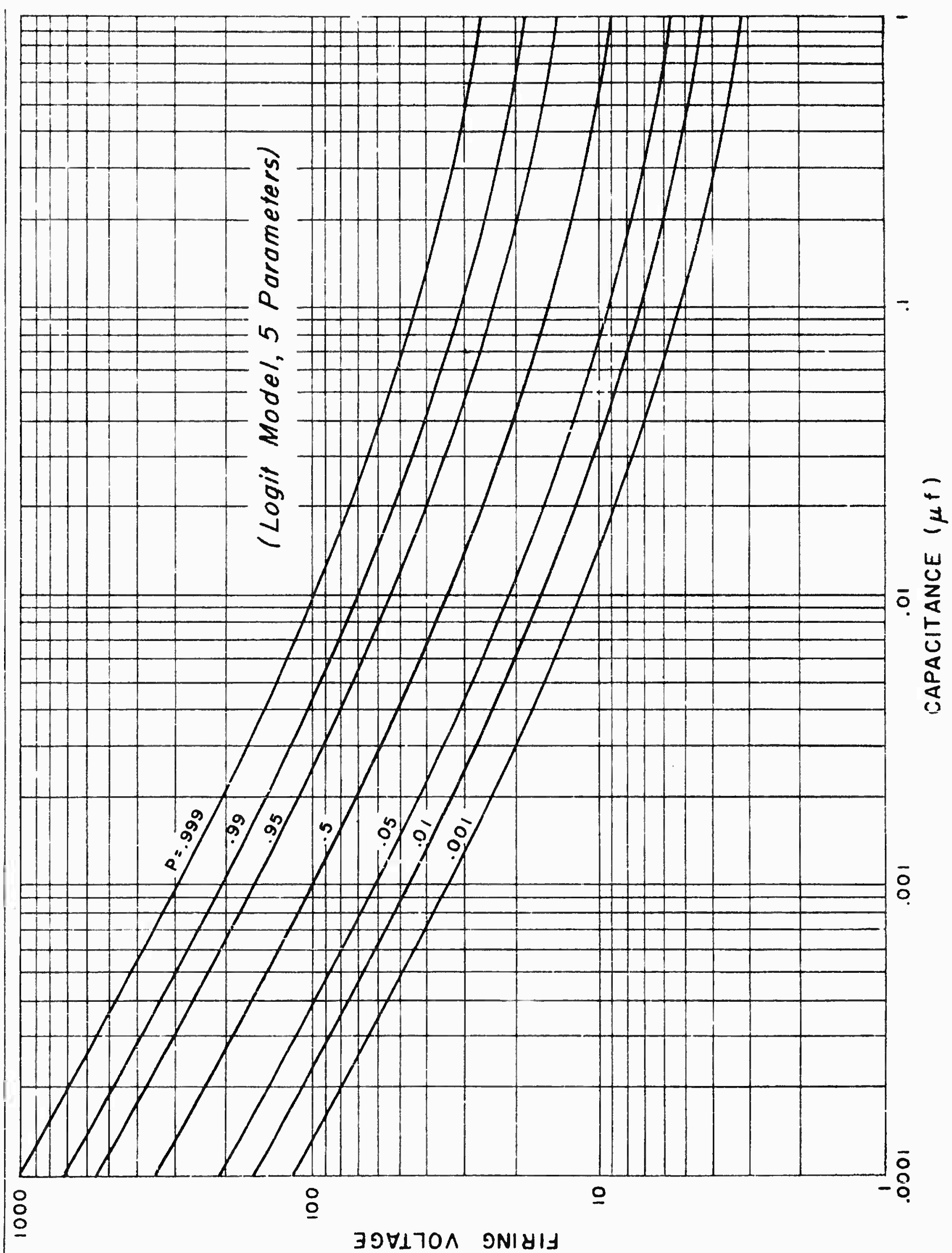
This model results in some variance between observed and estimated data. The variance may be compared with that obtained for the simpler model. If the variance reduction is insignificant, the introduction of the parabolic parameter c_{22} was unnecessary with a high degree of probability. If the variance reduction is significant, additional parameters may be introduced and tested in a like manner. Details of the computational procedure entailed in such a process are given in Appendix C.

Because of the excessive amount of computational work, the most refined model that was computed entirely was (20.3).

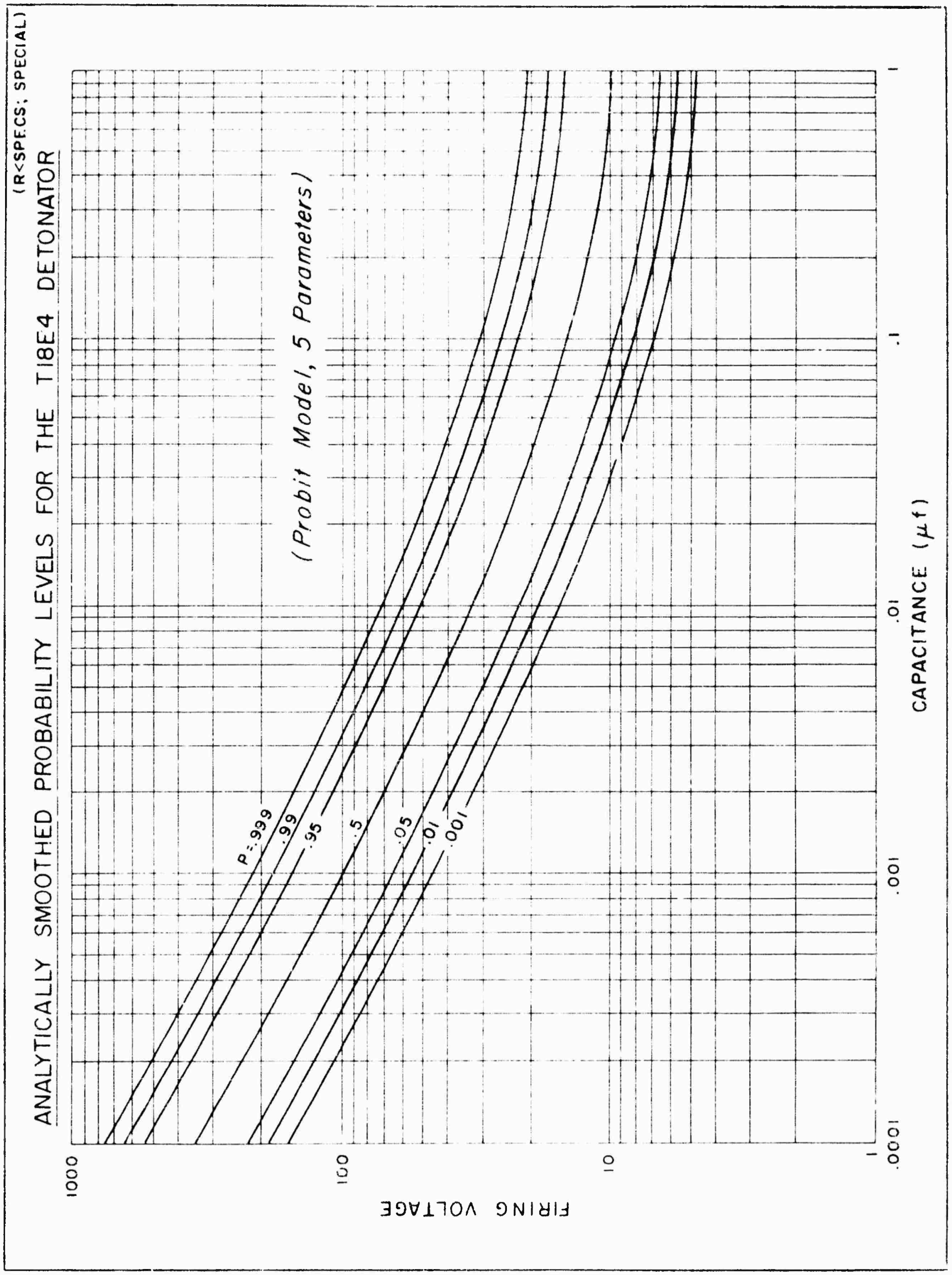
$$F_3(P) = a_3 + b_3 \log V + c_{31} \log C + c_{32} \log^2 C + c_{33} \log^3 C \tag{20.3}$$

Details of this work are outlined in Appendix C. By assigning probability levels P and choosing a sufficient number of points, we obtained Figures 4-8 and 4-9 for the logit and probit type analyses, respectively. We shall now compare these two figures with Figure 4-7 which was obtained by graphical smoothing.

First, we notice that the probability lines for P = 0.5 agree remarkably well. This agreement can be expected, of course, since the



ANALYTICALLY SMOOTHED FUNCTIONING PROBABILITY LEVELS FOR THE T18E4 DETONATOR
(R<SPECS; SPECIAL)



REPORT I-1804-1 FIGURE 4-9

Bruceton tests used in Figure 4-7 yield extremely good estimates of the mean firing levels, and since the two analytical methods are designed to yield efficient estimates for the same mean firing levels.

Next, we notice that the dispersion, or the spread between the various probability lines is least for the graphical analysis, larger for the analytical probit analysis, and largest for the analytical logit analysis. Good reasons can be stated for this behavior of the extreme probability lines. These reasons have to do with the intrinsic properties of the several models. Beginning with the Bruceton-graphical model, we know that the "local" sigmas are computed from a formula which includes all-fire and all-no-fire levels. Such a procedure tends to give smaller sigmas than a computational scheme which does not include the extreme levels. However, that is just the point which we want to make. Both the logit and probit type analyses cannot make use of these extreme firing levels and, therefore, tend to yield larger estimates of the sigmas. This tendency of the two analytical methods is enhanced by the fact that the analytical procedure does not give individual sigmas. It smoothes them in a separate operation (as we have done in equation (17)); but the analytical procedure gives an estimate of an over-all sigma from the over-all pattern of experimental points.

These observations are not arguments for or against the use of either procedure. They are merely observations of properties inherent in the use of any one of these models. A decision as to which model represents the data "best" cannot possibly be made on such short order, especially since we have accumulated little experience with analytical models. However, they are facts of which the user of such models should be aware and we shall point out some of the consequences in Section 5.

4.7 Resumé

The actual functioning or firing of an initiator is an attribute which can be measured as yes-or-no quantal response for a given set of inputs. Unavoidable variations in the manufacture of initiators and uncontrolable effects of instrumentation used in the tests cause one unit to function for a set of inputs, while another unit malfunctions under the same input conditions. In order to depict the functioning probability of detonators, a mathematical model has been constructed which states that the functioning probability is dependent on the (logarithm of) firing voltage and the (logarithm of) firing capacitance: $P = P(\log V, \log C)$, where P is the probability of firing. The solution of this equation leads to lines with constant firing probability in a log V-log C coordinate system. These lines can be obtained from a graphical procedure, described in Section 4.5 of this report. The basic data needed for this graphical procedure are Bruceton, probit, or Bartlett type tests at a sufficient number of levels for the firing capacitance. The evaluation of such data may also proceed according to several types of analysis and the relative merits of these analyses are discussed in detail. All methods studied here yield efficient estimates of the mean firing level, but the measures of dispersion estimated by the various analyses differ from one another. The logit type analysis yields the widest probability bands while probit and Bruceton type analyses yield narrower tolerance level bands. Analysis of the only Bartlett type test available thus far indicates that the logit type analysis may fit the detonator data significantly better than the probit type model. It also confirms a high degree of skewness, if duds are present or suspected.

The graphical procedure is supplemented by analytical methods suitable to obtain the firing probability curves with any desired degree of precision. Analytical evaluation of multivariate quantal response data involves a great amount of numerical computation. It seems hardly feasible to carry out such analyses on a routine basis unless high speed electronic computers, such as the UNIVAC, are employed.

Confidence bands for the tolerance levels estimated graphically or analytically can be found from the pertinent equations shown in Appendix B. These confidence bands are rather wide as long as the number of units available for testing is limited. However, by a proper choice of the test levels and the models, e.g., by use of very refined experimental designs, these confidence bands can be compressed to a minimum such that extreme firing levels can be estimated with greater confidence than at the present time.

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5. APPLICATIONS OF FUNCTIONING TIME AND SENSITIVITY ANALYSES

The methods outlined in the preceding sections are essentially descriptive and furnish a detailed analysis for any given initiator. Given a sufficiently large sample, the confidence bands providing estimates prepared from this description will be narrow. Although there remain many unanswered questions, we can use effectively the information already gained. In this section we shall discuss briefly some of the possible uses for this information. The inclusion of these illustrative examples may not only clarify the interpretation of our results but also pave the way for future work.

5.1 Specifications

The mass production of initiators involves repeated testing. This procedure assures the "consumer" of the uniform quality of the product with controlled variations from the norm. No manufacturer can produce units that are identical. Limited variations in quality are therefore controlled by inspection and testing which should cover the whole range

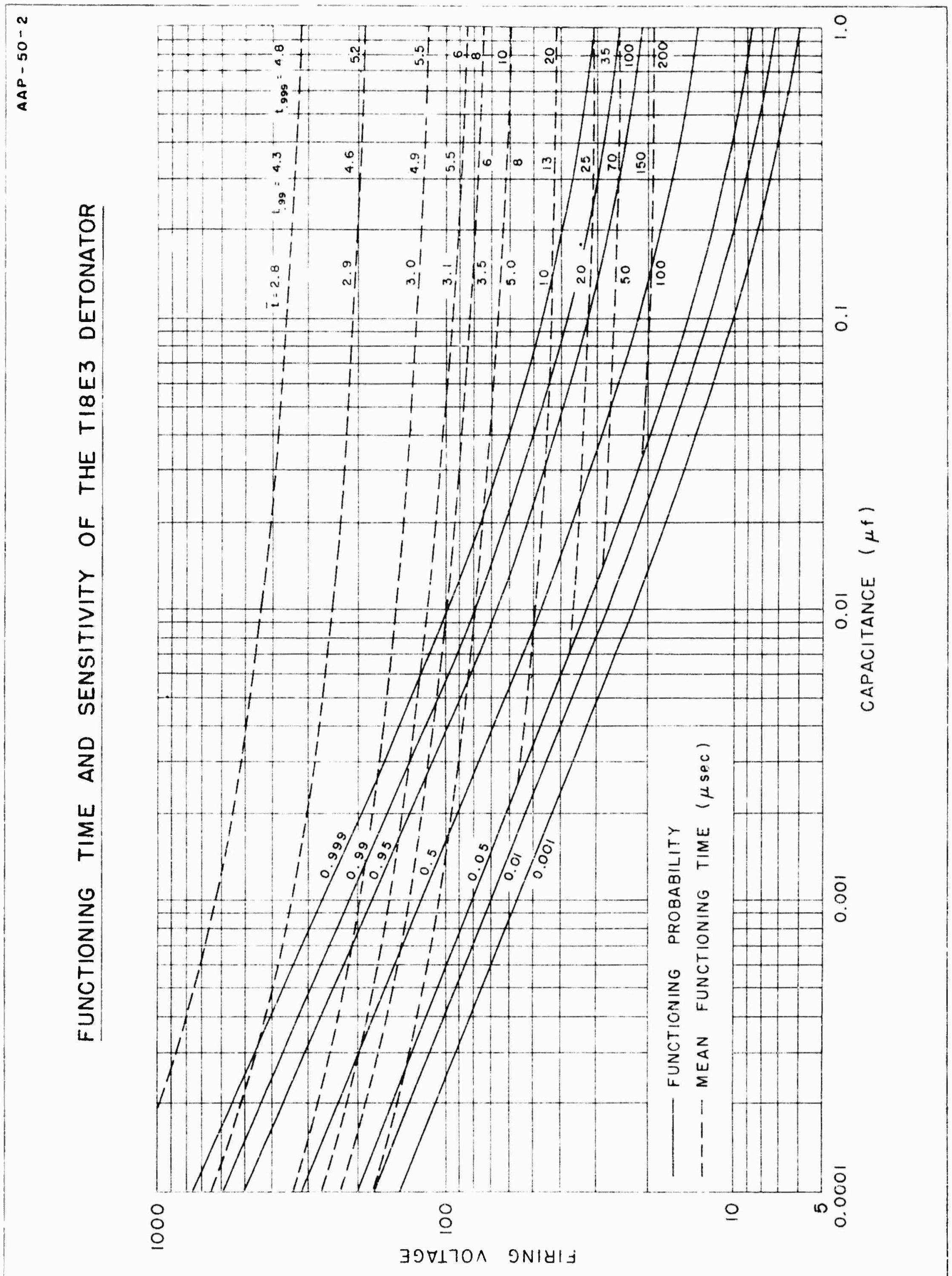

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FIGURE 5-1

accepted regularly as they meet this specification. However, there is no guarantee that the accepted detonators have uniform characteristics. For example, the sensitivity of the detonator could have undergone drastic changes at firing capacitances other than 1.0 microfarad without our ever finding this out from the proposed specifications. The diagram shows that the functioning time will be less than 10 microseconds with probability 0.999 when firing with 80 volts from 0.01 microfarads, but changes in the manufacturing process might cause another lot to have greater functioning times than 10 microseconds at this new test level. Such changes will escape us if we use just one specification test level.

Another feature is introduced by the use of MIL-STD-105A. If we plan to test detonators at the 0.999 probability firing level, enormous quantities are needed to complete one test. Since fewer units are needed to test at lower probability levels, what is the advantage of testing at the higher levels? This question drives at the heart of the entire specification test program and cannot be answered by a simple statement.

First, we should determine what are the reasons for testing at a given probability level. If the chief reason is <u>utmost</u> reliability, then the high probability test level is justified; but a large number of units must be expended in the process. If the chief reason is the need for a descriptive measure, then testing at a lower level becomes admissible and entails a reduction in expended items. If we attach a risk function to each test level and assign some measure to the risk which we are willing to take, then an optimum test level can be found by applying modern principles of statistics and probability such as those used in demand analysis and sampling surveys.

5.1.2 Sensitivity Specifications

Functioning time tests led us (Fig. 5-1) to investigate levels ranging from 60 to 80 volts for 1.0 μf capacitance. Similarly, we may determine levels for testing sensitivity or functioning probability. For example, if the functioning probability of 0.999 is to be tested, the proper voltages are 42 volts at 1.0 microfarads, or 120 volts at 0.01 microfarads. Additional large quantities of detonators are needed to perform these sensitivity tests. However, the test levels for sensitivity and for functioning time can be combined. If we search for the intersection of the line $P = 0.999$ with the line $t_{.999} = 10$, we find a suitable test level for the combination of both tests: $V = 70$ volts, $C = 0.045$ microfarads. This point appears to be suitable for the purpose of writing one-sided acceptance specifications. If the characteristic curves, shown in Figure 5-1, do not change materially during continued mass production, the over-all response pattern of the detonator can be inferred with 99.9% probability from the response at this point.

Finally, we must consider the problem of two-sided test specifications. The foregoing tests have established only that the detonator will almost certainly function under definite input conditions. However, manufacturers might abuse one-sided specifications by making their product ultra-sensitive. If the probability curves were shifted far below the required specification level, detonators might become so sensitive that they would no longer be safe to handle. Accidents from over-sensitivity , therefore, can be prevented by putting into effect a lower limit on sensitivity. This can be done by specifying a no-fire test level. Figure 5-1 shows that $C = 1.0$ microfarad and $V = 6$ volts might be such a test level. Together with the all-fire test level, two-sided specifications would guarantee sensitivity and safety, thus protecting producer and consumer alike.

5.2 Acceptance Sampling

Of the many articles and books written on the subject of acceptance sampling the most important for our purposes, is one issued by a Government agency. We refer, of course, to Military Standard, Sampling Procedures and Tables for Inspection by Attributes, MIL-STD-105A, dated 11 September 1950. This volume contains all the information needed here. Acceptance sampling plans are outlined in general, but they apply at once to the testing of initiators.

The "consumer" keeps a constant check on the "producer". He wants to be sure that the product he gets of uniform quality and conforms to specifications. Initiator specifications apply to a response probability of P = 0.999. The consumer will be satisfied if, in the long run, 0.1% of all initiators fail to respond at this particular level. This so-called "fraction defective" will not change if production has attained a stable state. The production line techniques agree then with those used to establish the unit's characteristic.

Two important features of all acceptance sampling plans are pertinent to our detonator problem. They are Sampling Plans and Severity of Inspection.

5.2.1 Sampling Plans

There are three types of sampling plans, namely, single inspection plans, double inspection plans or multiple inspection plans. (For details see MIL-STD-105A, Sect. 10, p. 3).

Single inspection plans have the advantage of simplicity. Only a certain number of units must be tested. These tests lead at once to a decision concerning acceptance or rejection of the whole lot. A disadvantage of single inspection plans is the large number of units required. If the lots range from 8001 to 22,000 units, Table III, page 10 of MIL-STD-105A shows the proper code letter to be "N". The acceptable defectives level of 0.1% corresponds to a functioning probability of 0.999. MIL-STD-105A, Table VI-N, page 44, specifies a sample size of 300. The lot is acceptable if at most one unit in the sample misfires. If more than one unit misfires, the lot must be rejected.

The same reference page shows that double sampling plans permit acceptance of the lot if a first sample of 200 contains no defectives.

In many instances, therefore, fewer than the 300 units required in the single plan will be needed. If up to 2 units misfire in the first sample, another sample of 400 ("Cumulative size 600") must show 100% response or the lot is rejected. It can be shown that the average number of units tested under this plan in the long run is LESS than 300! Hence it is more economical than the single sampling plan. The same reference table shows also the intricate details of a multiple sampling plan. Samples of 75 are taken and tested. Acceptance of the lot requires at least three such samples with a cumulative sample size of 225 for 100% response. The average number of units tested is again SMALLER than under the double sampling plan.

The consequences of any sampling plan are easily computed from its probability distribution. Details of these calculations are given in Appendix D of this report. First of all, no plan can effectively protect both consumers and producers from making an occasional mistake. Such a plan is impossible; it would require full inspection. But in destructive testing full inspection is not feasible. Even under the best plan, therefore, a "bad" lot slips through now and then and is accepted by the consumer although it should have been rejected. Occasionally, the consumer will also reject a perfectly "good" lot just because the probabilities work against him.

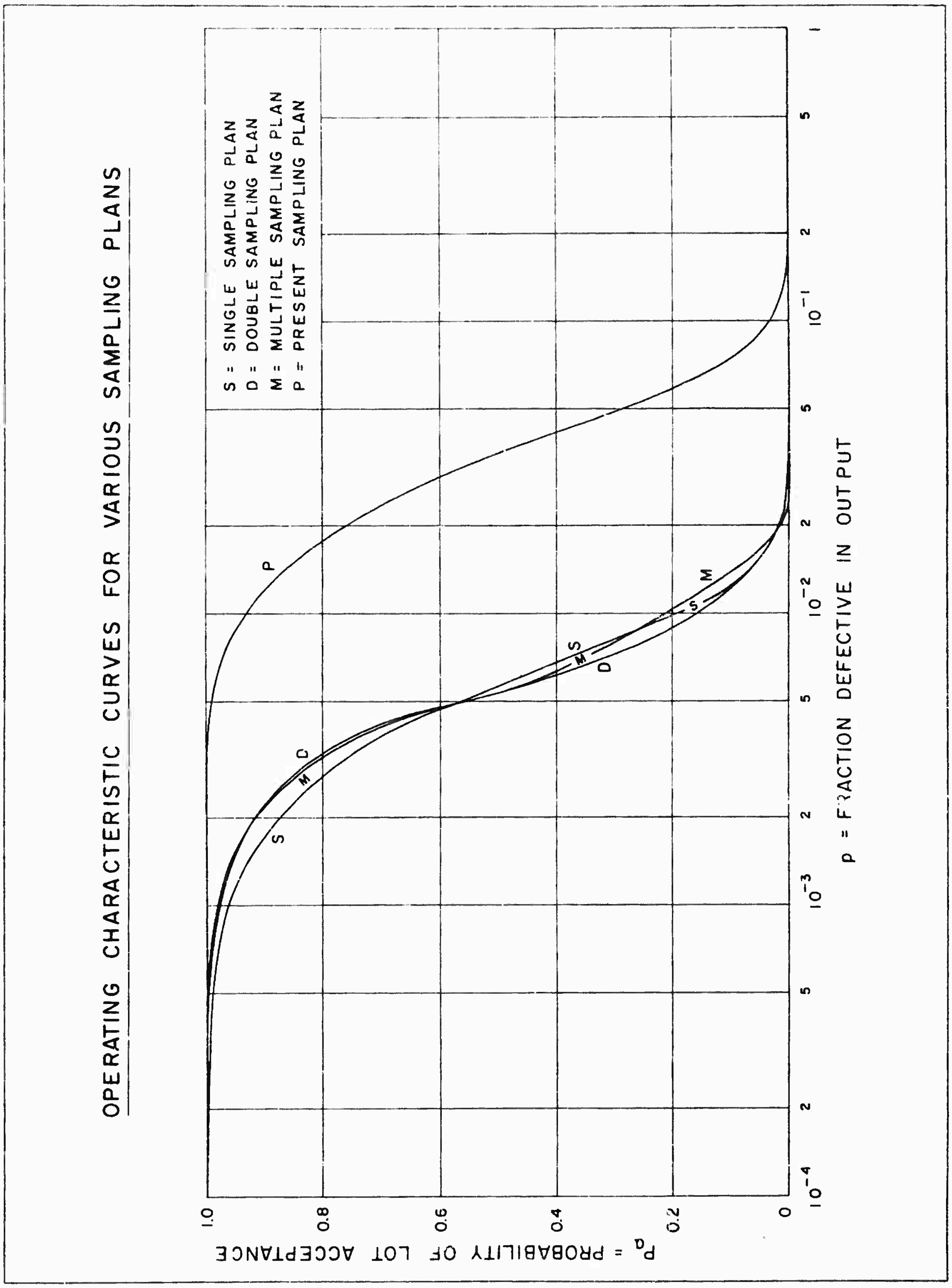
Such decisions create the risks taken by one or the other party. The risk of accepting a "bad" lot is usually referred to as the consumer's risk. The risk of rejecting a "good" lot is usually called the producer's risk. These risks are held low in any fair sampling plan and depend very much upon the acceptable quality level. In our case, the initiator output is acceptable if it contains 0.1% defectives. Now, if the production line suddenly deteriorates and produces lots that are 1% defective, the sampling plan should quickly catch such a change in quality. But if the output sinks to a level of only 0.2% defective, the plan cannot be expected to react quickly. The very first lot of this nature will probably not be rejected.

Figure 5-2 compares consumer's and producer's risk for several plans. The figure is drawn on a scale different from that used in MIL-STD-105A. It emphasizes the 0.1% defective acceptable quality level through the use of a logarithmic scale. The three plans agree well in the risks incurred. The fourth curve refers to one of the present acceptance sampling plans. This plan requires the testing of 50 units and permits retesting of 100 units if two units fail in the first sample. The retest must not show any failures. It is easily seen that this plan offers no protection whatsoever. "Bad" lots with 0.5% defectives will hardly ever be rejected: the probability for rejection is only one percent. Lots slightly better are probably always accepted. The plan does not operate at the quality level of 0.1% defective. It works nearer a level of 0.65% as shown by the corresponding column of Table VI-J, MIL-STD-105A. Of the various plans only those of MIL-STD-105A should be seriously considered for testing of initiators. The present plan discussed above does not protect the consumer.

The cost of each plan depends on the average number of units tested in the long run. Let us assume that production has been in full swing for some time (steady state) and that the outgoing quality level stands at 0.1% defectives. Then the following tabulation describes the average cost of each plan in terms of units tested (App. D) as follows:

Sampling Plan	Single	Double	Multiple	Present
Average Number of units tested	300	272.1	260.8	50.1

The table confirms the well-known fact that the multiple sampling plan is always the most economical. The present plan uses up fewer units on the average than any of the other plans; but this merely reflects the fact already stated that it offers no protection. On the surface, the savings of the multiple over the single plan appear small. But during



REPORT I-1804-1 FIGURE 5-2

mass production, samples are taken weekly, or even daily. A saving of even fifty detonators per sample represents a sizable investment on the part of the consumer, over the period of one year. Further savings are possible through relaxed inspection methods (Section 5.2.2).

5.2.2 Severity of Inspection

Acceptance sampling from a production line may have continued for a while under a specific plan. After a year or so, it may come about that the production line operates at a uniformly stable level and that product quality conforms to the acceptable per cent defective. Why, then, should the tests continue on as severe a basis as was described in Section 5.2.1?

Statisticians have coped with this problem in two ways. They recommend that the initial sampling plan be very severe. This establishes confidence in the product quality. Following a brief period of severe inspection, a longer period of normal inspection is recommended. This insures stability of the production line output. Finally, inspection may be relaxed. Only a few token units are tested from each lot. As long as they conform to certain cumulative standards, relaxed inspection may be continued.

Tightened inspection (MIL-STD-105A, p. 44) requires that samples of 100 be taken with the multiple plan; samples of 300 and 600 with the double sampling plan; and a sample of 450 with the single sampling plan. After a number of lots have been inspected, we may return to normal inspection, if there were sufficiently few misfires. The normal inspection plan is now put into effect. After some time, many samples and lots have been tested. Their cumulative totals may permit the use of reduced inspection. Table II tells us when to begin reduced inspection. Table V shows that for lots of size "N" the sample size may be reduced to 60 units per lot as long as inspection shows that the output conforms to Table II.

This reduction in sample size is possible only through the continuous calculation of cumulative totals tested and found defective. As soon as these cumulative totals exceed the limits shown in Table II, tightened inspection is at once employed until the production process has again attained a normal level. These conditions are embodied in the final sampling plan, proposed in Section 5.2.3.

5.2.3 Acceptance Sampling Plans for Initiators

The plan suggested here for the testing of initiators is based upon standards approved by the Armed Forces and by the Chairman of the Munitions Board. MIL-STD-105A states on page II the following:

1. This Standard is approved by the Departments of the Army, the Navy, and the Air Force for the purpose of establishing sampling plans and procedures for inspection by attributes.

2. This revision supersedes JAN-STD-105, dated 15 February 1949, as amended, Sampling Inspection Tables for Inspection by Attributes.

3. The Quartermaster Corps, USA; the Bureau of Ships, USN; and the AMC, USAF, are designated custodians of this Standard.

4. This Standard is mandatory for use by the Departments of the Army, the Navy, and the Air Force when applicable. Deviations found necessary when actually using this Standard shall be reported to the Munitions Board Standards Agency.

5. Recommended corrections, additions, or deletions should be addressed to the Chairman, Munitions Board Agency, Washington 25, D.C.

Therefore, it is recommended that the testing of initiators be subject to tightened, normal, or reduced inspection as defined in MIL-STD-105A. A minor refinement suggested here is the use of semi-logarithmic paper for the risk curves and two-sided specifications. With the curves, the user can study in greater detail the operation of the plans at very high quality levels, e.g., at a fraction defective of 0.1% or less. Test levels for initiator tests should reflect both the functioning time and the functioning or response probability of P = 0.999 and of P = 0.001. Purchase specifications for the T18E3 might be written with such requirements in mind. As the present sampling plan offers relatively little protection to the consumer, it should be modified in accordance with these principles embodying (a) MIL-STD-105A (b) two-sided specifications.

Such a procedure offers a number of advantages not yet brought out in the discussion. Not only does it offer protection, not afforded by the present plan, to both producer and consumer, but it permits also a control of improved or improvable manufacturing conditions. The reason advocated thus far for using two-sided specifications is the desire to achieve both safe lower limits and sure firing limits. However, as production methods improve, they may become standardized and possibly even mechanized. It is quite likely that the performance of detonators can then be improved so as to approach more closely the ideal curve shown in Figure 4-1. If such a shift towards an improved product takes place, the two-sided specifications can be used to "squeeze" manufacturing techniques towards standards already achieved by many other industries. In addition, applications are being studied, where the lower limit of detonator performance is vital to the effective functioning of some device. Slow voltage buildup used in the arming of mines, and firing of several detonators from a parallel circuit and a variable source are two such examples.

5.3 Comparison of Detonators

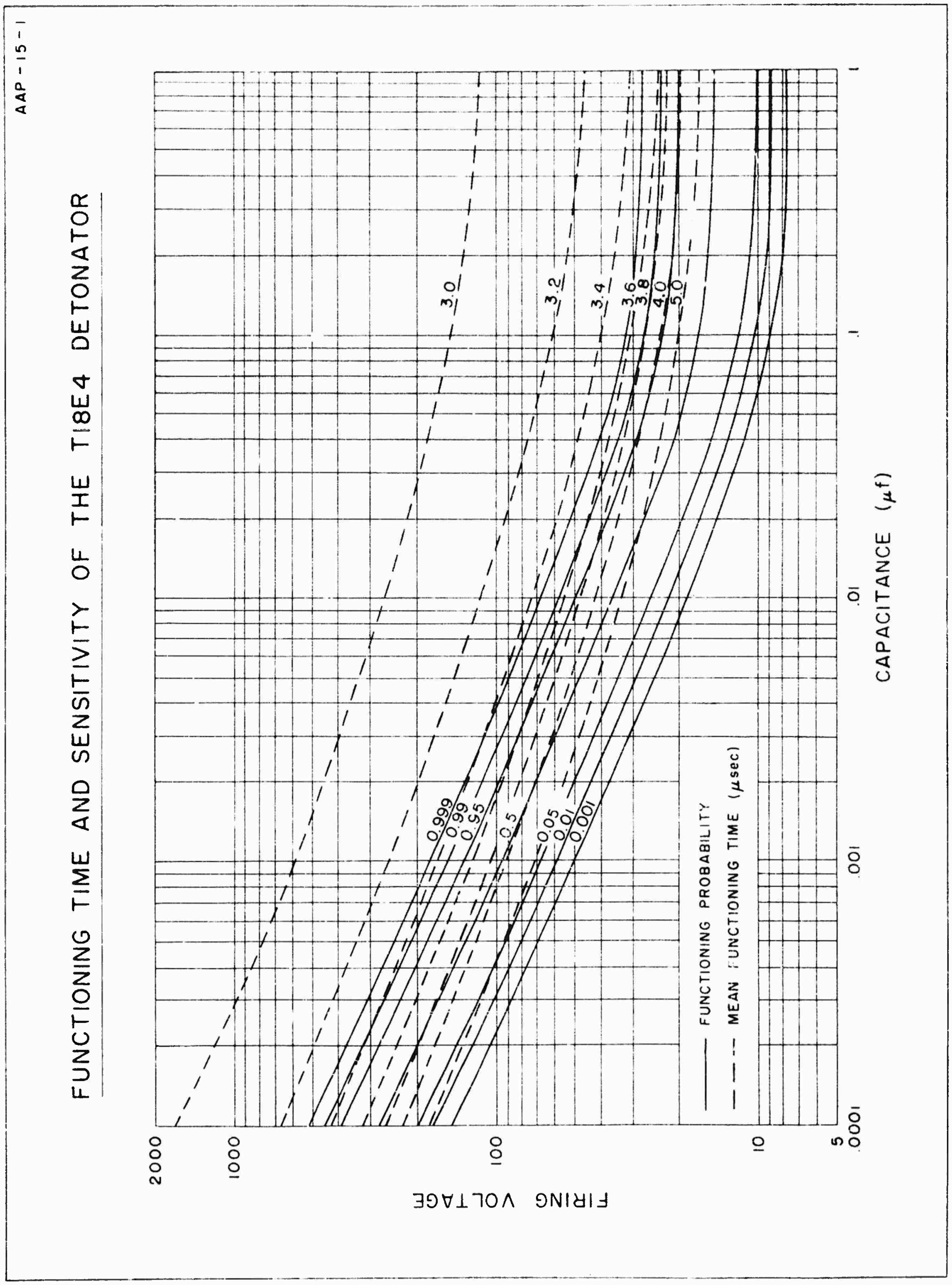
This project has afforded us an opportunity to test and evaluate a large number of detonator types and lots. In so doing, we have made two rather remarkable discoveries which influenced our thinking and may have some bearing on future developments in the detonator industry.

First, we have analyzed a large number of lots belonging to the same type of detonator. For example, in addition to T18E3 (AAP-50-2),

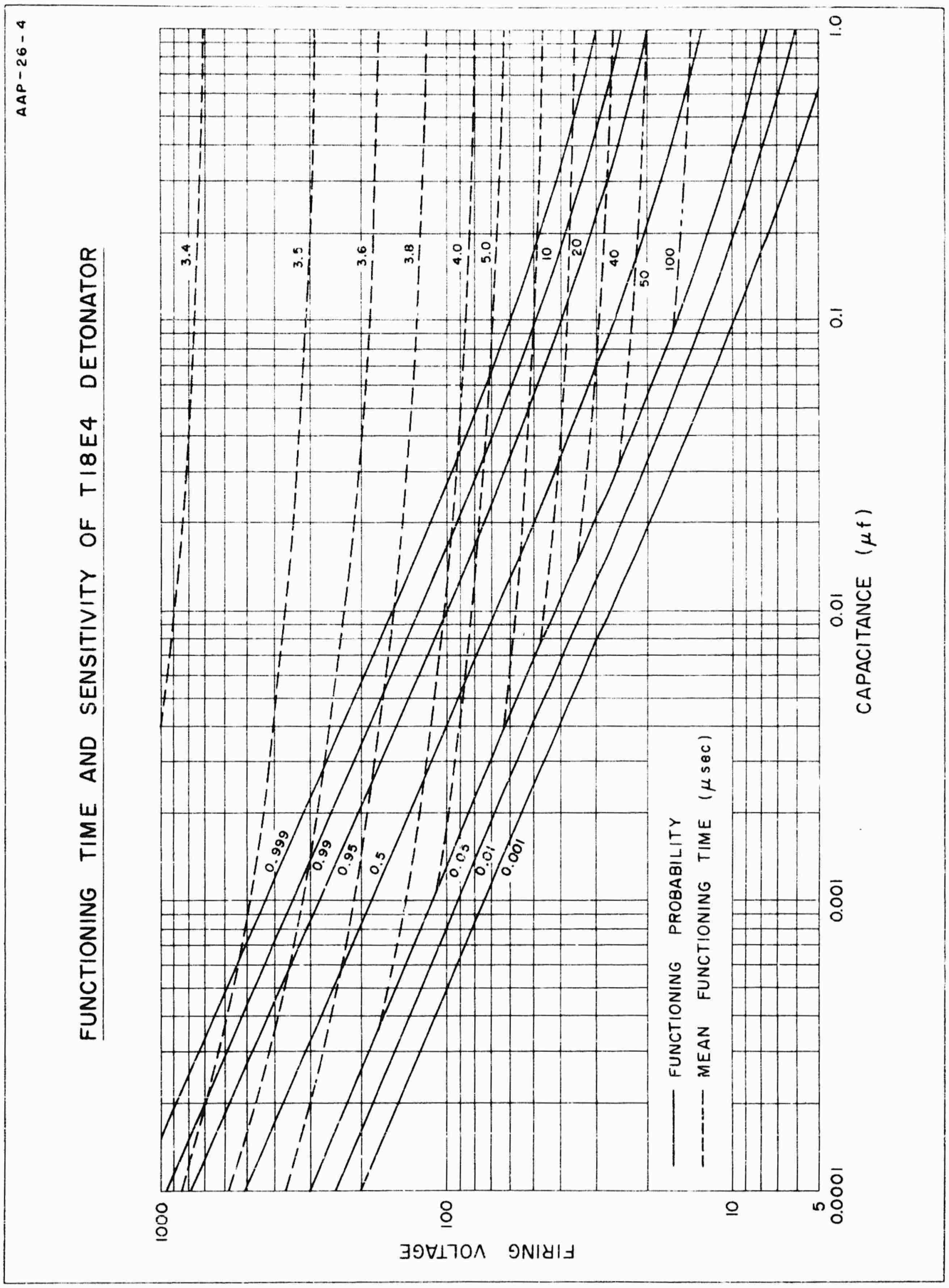
Figure 5-1, we have evaluated T18E4 (AAP 15-1) and T18E4 (AAP 26-4) carbon bridge detonators. The two resultant families of curves are shown in Figures 5-3 and 5-4. It is at once apparent that these last two detonators cannot possibly come from the same population, although they were both represented as T18E4 detonators. Sampling variations may account for a small part of the differences, but statistical tests have convinced us that the last two lots are significantly different in many respects with a high degree of probability. In other words, production of detonators, frequently referred to as an art, yields one lot after another, each with different characteristics. There exists, of course, some over-all degree of uniformity, but upon close scrutiny it is difficult to detect uniformity in more than just the descriptive number assigned by the manufacturer. Little meaning can therefore be attributed to statements about detonator types unless their lot characteristics are stated or specified. Two lots of detonators are just not alike.

Second, the differences between two detonator lots of the same type are sometimes even larger than differences between one type detonator and another. Comparing Figures 5-3 and 5-4 for two lots of T18E4 with one lot of T18E3, Figure 5-1, we find the latter falls in many respects between the former. Of course, we could not picture all characteristics such as effects of temperature cycling, drop tests, humidity cycling, etc., but the fact remains that the differences between detonator types may not be any greater than between detonator lots.

These two discoveries have caused us to abandon the idea that detonator evaluation should yield population characteristics. Inferences as to population characteristics must be made from sample characteristics with extreme caution. Frequently they will result in extremely wide confidence bands for the estimated population characteristics. In view of these facts, it seems as if standardization of detonator production might be more readily achieved than is believed by many. If the characteristics of individual lots vary as much as is indicated by our studies, introduction and production of new types is meaningful



REPORT I-1804-1 FIGURE 5-3



REPORT I-1804-1 FIGURE 5-4

only if these new types are indeed different in their response. Many of the varieties used now do not seem to possess such radically different characteristics and do not warrant designations different from existing types.

Another, very interesting observation has been made during our continued study of various types of detonators. We noticed a very pronounced correlation between quantal response and functioning time which, in some instances was even independent of firing capacitance. If we look once more at Figure 5-4, we note that the larger functioning probabilities correspond to shorter functioning times, while the lower functioning probabilities occur together with longer functioning times. For this T18E4 detonator, the relationship between functioning probability and functioning time is also strongly influenced by the firing capacitance: the probability range $P = 0.5$ to $P = 0.999$ corresponds to a mean functioning time range $\bar{t} = 3.7$ to $\bar{t} = 3.4$ microseconds at $C = 0.0001$ microfarads; to a mean functioning time range $\bar{t} = 45$ to $\bar{t} = 7$ microseconds at $C = 0.1$ microfarads. For wire bridge detonators, the relation is sometimes so strong that it becomes capacitance-independent. For the M114(Lot 78) Wire bridge detonator, we observed mean functioning times of 250 microseconds at a functioning probability level $P = 0.5$; a mean functioning time of 80 microseconds at the functioning probability level $P = 0.999$ over the entire range of firing capacitances tested. Further work will be necessary, of course, to ascertain the exact nature of this correlation between the quantal response variable of functioning and the continuous variable of functioning time. If and when the nature of this correlation has been established, testing and evaluation of detonators may well enter an entirely new phase, since the testing of variables is much more easily accomplished than that of a quantal response.

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11. Naval Ordnance Standard Sampling Procedures and Tables for Inspection by Variables, NAVORD OSTD 80, A Bureau of Ordnance Publication, 8 September 1952.

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6. SUMMARY

This report was written to summarize techniques developed and used at the Franklin Institute Laboratories for analysis and evaluation of initiator test data. The main tools, of course, are statistical in nature and the present study of initiator test data is concerned with functioning time and sensitivity. However, slight modifications of the tools presented here will permit evaluation of other factors, such as mechanical or electrical construction, pre-sparking of carbon bridges, temperature cycling, testing of stab type or conductive mix detonators, and many others.

One of the fields reported here is the study of the functioning time of initiators, a stochastic variable. We made use of the relationship, or model, $t_p = t_p(V, C, P)$ which states that the functioning time of an initiator remains below t_p with probability P if the initiator is fired from a capacity C at voltage V. The parameters which appear in this relationship can be found from experimental data, by a graphical or analytical procedure, and a suitable experimental design. These parameters may then be used to construct graphs which form part of the so-called EIH curves and depict the sample characteristics of the detonator lot tested. Extension to lot characteristics is feasible, but in view of our experience with changes from lot to lot, these lot characteristics are still not very meaningful. Functioning time graphs can be used to specify suitable acceptance sampling test levels, to determine operating characteristics for the user, and many other interesting applications.

The sensitivity of initiators is an observable attribute, or quantal response, which is also a stochastic variable and a function of the inputs. We made use of the functional relationship $P = P(V,C)$ which states that the probability of initiator response is a function of the firing capacitance C and the firing voltage V. The parameters which appear in the relationship are found with the help of various more or less elaborate experimental designs. Among them are the probit,

Bruceton, and Bartlett designs, each of which is augmented by an appropriate form of analysis. From the estimated parameters, graphs of the functional relationship $P = P(V,C)$ can be constructed by the use of either graphical or analytical techniques. These graphs form part of our EIH curves and depict the sample characteristics of the detonator test lot under study. Extension to lot characteristics is again possible, but yields very wide confidence bands for the estimated extreme functioning probability levels because of the large variations existing from one lot to the next. The sensitivity plots thus obtained can be used to find specification test levels, characteristic operating curves, and other interesting information.

The methods outlined in this report are general and are well known to statisticians. Yet, it was felt that a summary presentation of these methods would serve those engaged in testing and evaluating initiators by pin-pointing some of the difficulties which beset the experimenter. It is also hoped that this report will lead to a standardization of analytical techniques and provide a stimulus for at least two future studies of initiator characteristics. One phase of this future work concerns the true nature of the sensitivity curve, its non-normality, and the decomposition of sensitivity analysis into detonator and dud analysis. The other, even more intriguing phase aims to remove the quantal response nature from the evaluation procedure by proposing the existence of a correlation between functioning time and functioning probability. If such a correlation can be proved to exist, testing of detonators will assume much simpler aspects. If this report stimulates thinking in those directions, it will have served its purpose and will have been well worth the effort.

This report was prepared by Dr. Carl Hammer, Senior Staff Engineer, Analysis Section, Electrical Engineering Division and was reviewed by Irving Schwartz, Engineering Physics Section.

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APPENDIX A

COMPUTATIONAL PROCEDURE FOR FUNCTIONING TIME ANALYSIS

APPENDIX A

COMPUTATIONAL PROCEDURE FOR FUNCTIONING TIME ANALYSIS

It is the purpose of this appendix to describe the computational procedure used to derive the parameters which occur in the mathematical models for detonator functioning times. The model, described in Section 3.4, is written in logarithmic form:

$$\tau = a_{00} + a_{10} \log V + a_{01} \log C + a_{20} \log^2 V + a_{11} \log V \log C \qquad (21)$$
$$+ a_{02} \log^2 C + \ldots$$

The fitting procedure is the same, no matter how many parameters are chosen. We shall list here only the equations arising from one specific case and leave it to the reader to construct additional examples of greater, or lesser, complexity. We shall use a fitting which requires 5 parameters:

$$\tau_5 = a_{00} + a_{10} \log V + a_{01} \log C + a_{20} \log^2 V + a_{11} \log V \log C \quad (22)$$

The corresponding least squares equation, written in matrix form is

$$\begin{bmatrix} N & \Sigma \log V & \Sigma \log C & \Sigma \log^2 V & \Sigma \log V \log C \\ \Sigma \log V & \Sigma \log^2 V & \Sigma \log C \log V & \Sigma \log^3 V & \Sigma \log^2 V \log C \\ \Sigma \log C & \Sigma \log C \log V & \Sigma \log^2 C & \Sigma \log C \log^2 V & \Sigma \log V \log^2 C \\ \Sigma \log^2 V & \Sigma \log^3 V & \Sigma \log C \log^2 V & \Sigma \log^4 V & \Sigma \log^3 V \log C \\ \Sigma \log V \log C & \Sigma \log C \log^2 V & \Sigma \log V \log^2 C & \Sigma \log^3 V \log C & \Sigma \log^2 V \log^2 C \end{bmatrix} \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{20} \\ a_{11} \end{bmatrix} =$$

$$= \begin{bmatrix} \Sigma \tau_5 \\ \Sigma \tau_5 \log V \\ \Sigma \tau_5 \log C \\ \Sigma \tau_5 \log^2 V \\ \Sigma \tau_5 \log V \log C \end{bmatrix} \qquad (23)$$

The variance for this fitting $V\{\tau_5\}$ is obtained from

$$(N-6) \quad V\{\tau_5\} = \Sigma\,\tau^2 - a_{00}\,\Sigma\,\tau - a_{10}\,\Sigma\,\tau \log V$$

$$- a_{01}\,\Sigma\,\tau \log C - a_{20}\,\Sigma\,\tau \log^2 V$$

$$- a_{11}\,\Sigma\,\tau \log V \log C \qquad (24)$$

The Null-Hypothesis is rendered by

$$\tau_1 = a_{00} = \frac{\Sigma\,\tau}{N} \qquad (25)$$

and its variance $V\{\tau_1\}$ from

$$(N-2) \quad V\{\tau_1\} = \Sigma\,\tau^2 - a_{00}\,\Sigma\,\tau \qquad (26)$$

Table A-1 cont[illegible] basic data used to construct Figures 3-5, 3-6, and 3-7. These data apply to the T18E4 (R < Specs. Special) lot of carbon bridge detonators used in our Test 44. Table A-2 lists the summations obtained from these data, used to solve matrix equations (23). The solutions obtained from these matrix equations were shown in Table 3-5 together with the variances obtained from equations (24) above.

The method, as mentioned in Section 3.4 is perfectly general. It can be applied to any model of this nature. However, the solution of the matrix equations is tedious, even with the help of modern high speed electronic computing devices.

Table A-1. [illegible] (R < SPEC: SPECIAL) TEST 44 PP. 177-179,184 BASIC DATA

C	V	t	log C	log V	τ = log t
.001	149.6	3.250	-3.00000	2.17493	0.51188
		3.125			0.49485
	125.9	3.625		2.10003	0.55931
		3.125			0.49485
		4.000			0.60206
		3.250			0.51188
	105.9	3.875		2.02490	0.58827
		3.875			0.58827
		3.125			0.49485
		3.500			0.54407
		3.750			0.57403
		3.250			0.51188
	89.13	3.500		1.95002	0.54407
		3.500			0.54407
		3.500			0.54407
		2.750			0.43933
		3.250			0.51188
		3.250			0.51188
		3.500			0.54407
	74.99	3.625		1.87500	0.55931
.1	13.34	4.625	-1.00000	1.12516	0.66511
		4.750			0.67669
		16.125			1.20750
	15.85	7.000		1.20003	0.84510
		18.750			1.27300
		6.375			0.80448
		10.500			1.02119
		7.125			0.85278
		7.750			0.88930
		14.250			1.15381
		4.375			0.64098
		9.125			0.96023
		4.375			0.64098
	18.84	5.875	-1.00000	1.27508	0.76901
		6.000			0.77815
		4.500			0.65321
		5.000			0.69897
		4.500			0.65321
		5.125			0.70969
.0001	298.5	3.375	-4.00000	2.47494	0.52827
		3.625			0.55931
		3.625			0.55931
	354.8	3.500		2.54998	0.54407
		3.250			0.51188
		3.375			0.52827
		3.625			0.55931
		3.375			0.52827
	421.7	3.500		2.62500	0.54407
		3.375			0.52827
		3.500			0.54407
		3.00			0.47712
		3.00			0.47712
		2.875			0.45864
		3.000			0.47712
	501.2	3.125		2.70001	0.49485
.01	44.67	4.375	-2.0000	1.65002	0.64098
		4.500			0.65321
		4.000			0.60206
	37.58	4.625		1.57496	0.66511
		3.125			0.49485
		3.625			0.55931
		5.500			0.74036
		5.375			0.73038
		5.250			0.72016
		7.750			0.88930
		5.750			0.75967
		4.875			0.68797
	31.62	4.500	-2.0000	1.49996	0.65321
		7.000			0.84510
		3.500			0.54407
		7.500			0.87506
		3.875			0.58827

C	V	t	log C	log V	τ = log t
.01	31.62	5.500	-2.0000	1.49996	0.74036
		6.250			0.79588
		3.895			0.59051
1.0	1000	3.000	0	3.00000	0.47712
		3.000			0.47712
		3.250			0.51188
		3.250			0.51188
		3.375			0.52827
		3.125			0.49485
		3.125			0.49485
		3.250			0.51188
		3.375			0.52827
		3.500			0.54407
.01	1000	2.875	-2.0000	3.00000	0.45864
		3.250			0.51188
		3.125			0.49485
		2.750			0.43933
		3.250			0.51188
		3.625			0.55931
		3.375			0.52827
		2.750			0.43933
		2.750			0.43933
		2.750			0.43933
.0001	1000	2.875	-4.0000	3.00000	0.45864
		2.625			0.41913
		3.000			0.47712
		3.000			0.47712
		2.875			0.45864
		2.750			0.43933
		2.750			0.43933
		3.500			0.54407
		3.250			0.51188
		3.375			0.52827
.1	30.	14.625	-1.0000	1.47712	1.16510
		4.500			0.65321
		3.875			0.58827
		4.000			0.60206
		3.500			0.54407
		4.25			0.62839
		5.000			0.69897
		4.750			0.67669
		3.875			0.58827
		4.625			0.66511
1	100	3.500	0	2.00000	0.54407
		3.500			0.54407
		3.000			0.47712
		3.250			0.51188
		3.500			0.54407
		3.500			0.54407
		3.750			0.57403
		3.250			0.51188
		2.875			0.45864
		3.500			0.54407
.01	100	3.375	-2.0000	2.00000	0.52827
		3.250			0.51188
		3.625			0.55931
		3.250			0.51188
		3.375			0.52827
		3.125			0.49485
		3.625			0.55931
		3.375			0.52827
		3.625			0.55931
		3.375			0.52827
1.0	7.943	24.375	0	0.89998	1.38694
		22.125			1.34488
1.0	9.441	28.125	0	0.97502	1.44909
		47.250			1.67440
		13.625			1.13434
		26.125			1.41706
		18.875			1.27589
		11.000			1.04139
		15.125			1.17970
		35.500			1.55023

Table A-1. T18E4 (R < SPECS: SPECIAL) TEST 44 PP. 177-179, 184 BASIC DATA (Contd.)

C	V	t	log C	log V	τ = log t
1.0	9.441	25.750	0	0.97502	1.41078
		29.00			1.46240
		29.250			1.46613
1.0	11.22	10.125	0	1.04999	1.00540
		17.125			1.23363
		15.000			1.17609
		18.125			1.25828
1.0	13.34	17.125	0	1.12516	1.23363

Table A-2. SUMMATIONS OF LOGARITHMIC FUNCTIONING TIMES
T18E4 (R < SPECS: SPECIAL)

Σ log C	-273.00000
Σ $\log^2$ C	785.00000
Σ $\log^3$ C	-2553.00000
Σ $\log^4$ C	8945.00000
Σ $\log^5$ C	-32793.00000
Σ $\log^6$ C	123,665.00000
Σ log V	298.44639
Σ $\log^2$ V	654.26533
Σ $\log^3$ V	1570.18305
Σ $\log^4$ V	4021.30977
Σ log C log V	-606.31870
Σ $\log^2$ C log V	1866.11134
Σ $\log^3$ C log V	-6338.23726
Σ $\log^4$ C log V	22850.10454
Σ log C $\log^2$ V	-1437.91295
Σ $\log^2$ C $\log^2$ V	4642.86097
Σ $\log^3$ C log V	-16257.67109
$\Sigma\tau$	104.39383
$\Sigma\tau^2$	83.26401
$\Sigma\tau$log C	-154.83727
$\Sigma\tau\log^2$ C	423.58349
$\Sigma\tau\log^3$ C	-1338.88873
$\Sigma\tau$log C log V	-329.74429
$\Sigma\tau\log^2$ V	367.72251
Σ log C $\log^3$ V	-3594.02692
$\Sigma\tau$log V	182.21322
N	153.

APPENDIX B

SENSITIVITY MODELS FOR INITIATORS

APPENDIX B

Sensitivity Models for Initiators

In this note we shall compare two of the models that have been use extensively to describe the sensitivity of detonators at fixed firing capacities. The first model is based upon Berkson's Logit Analysis, the other on Finney's Probit Equations which are often approximated by the so-called Bruceton Equations. We shall point out some of the difficulties encountered in estimating extremely high or low firing levels.

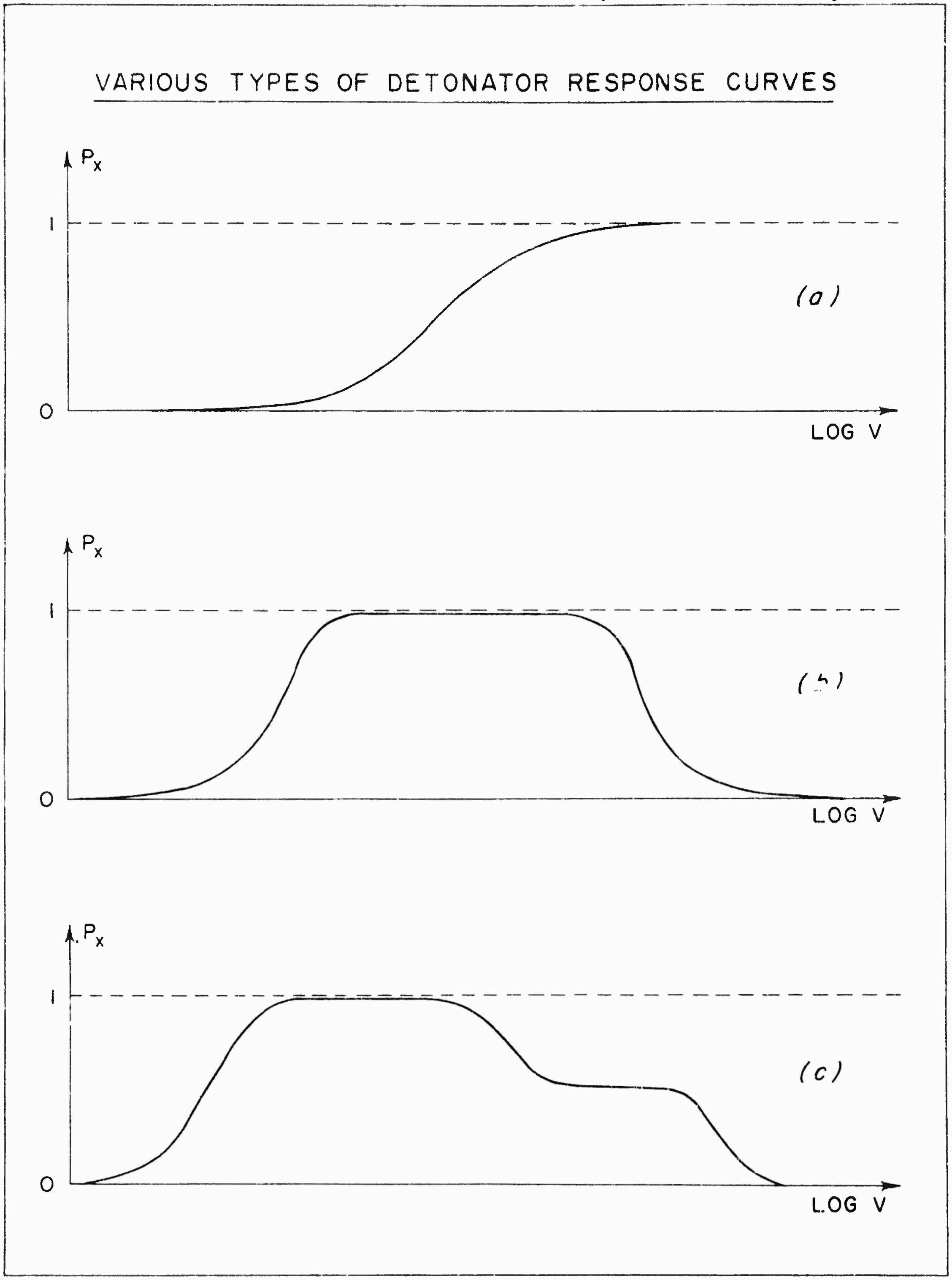
Figure B-1 shows a typical sensitivity curve for constant firing capacitance C and variable firing voltages V. The detonator response curve resembles a sigmoid function only accidentally and it is quite conceivable that it might even follow Figures B-1b or B-1c for special types of initiators. However, at the present time no one seems to know how to construct a physical initiator with a given characteristic curve such as that in Figure B-1a, much less with a response such as that in Figure B-1b or B-1c. Nevertheless, such response curves are known for other physical phenomena.

Our present state of knowledge implies simply that the firing probability and the firing voltage (or energy) increase together. But if some sort of filter action could be built into the detonator, it could be made to obey any type of complicated response. In the following, we shall assume that the response curve is a monotonically increasing sigmoid which approaches zero and unity, respectively, for extremely low or high firing voltages. It will be convenient to discuss the two models separately, remembering that both are strictly empirical as there does not exist an adequate physical theory to support either.

1. BERKSON'S LOGIT MODEL

The basic equation for this model is

$$\log\left(\Pi/(1-\Pi)\right) = \alpha_1 + \beta_1 \log V \qquad (27)$$



REPORT I-1804-1 FIGURE B-1

where the logarithms are natural logarithms. In actual computations, however, they are replaced by common logarithms and the modulus M. $0 \leqslant \Pi \leqslant 1$ is the true, unknown firing probability for voltage V. (α_1, β_1) are the true, unknown parameters in the model. Solving for V and Π, we have

$$V = V_\Pi = (\Pi/(1-\Pi))^{1/\beta_1} \exp(-\alpha_1/\beta_1) \tag{28}$$

$$\Pi = \Pi_V = 1/(1 + V^{-\beta_1} \exp(-\alpha_1)) \tag{29}$$

The fitting process is usually carried out with equation (27) since it permits a linear regression estimate of the desired parameters α_1 and β_1. The firing probability Π cannot be observed directly and the model is written in terms of observed fires n_x, failures n_o, and tested number of initiators $n = n_x + n_o$. The observed relative frequencies $n_x / n \approx P$ and $n_o / n \approx Q = 1 - P$ are estimates of Π and $1 - \Pi$, while (a_1, b_1) are estimates of (α_1, β_1) :

$$\log (P/Q) = a_1 + b_1 \log V. \tag{30}$$

Estimation of the parameters by the maximum likelihood principle yields least squares equations which must take into account the number n of units fired at each voltage level V. Several conventions for assigning appropriate weights have been employed. Indicating the weights simply by w, we may use

$$w_1 = n_o n_x / n \approx n P Q \tag{31}$$

$$w_2 = n \tag{32}$$

$$w_3 = 1 . \tag{33}$$

Each of these weights has some statistical justification and the least squares equations become now

$$a_1 \Sigma w + b_1 \Sigma w \log V = \Sigma w \log (n_x/n_o) \qquad (34)$$

$$a_1 \Sigma w \log V + b_1 \Sigma w \log^2 V = \Sigma w (\log V) (\log (n_x/n_o)) \qquad (35)$$

The solution of these equations yields the desired estimates of the model parameters with which the estimated number of fires and failures can be computed:

$$N_x = n / (1 + V^{-b_1} \exp (- a_1)) \qquad (36)$$

$$N_o = n - N_x \qquad (37)$$

For comparison with other models, we have the variance in the number of fires

$$U_1 (w) = k \Sigma w (n_x - N_x)^2 / (k-2) \Sigma w) \qquad (38)$$

More important is the variance of the fittings from

$$U_1(w) = k \Sigma w (\log (n_x/n_o) - \log (N_x/N_o))^2/((k-2) \Sigma w) \approx \sigma^2 \qquad (39)$$

$$= k [\Sigma w \log^2 (n_x/n_o) - a_1 \Sigma w \log (n_x/n_o)$$

$$- b_1 \Sigma w (\log (n_x/n_o) (\log V)] /((k-2) \Sigma w) \qquad (40)$$

This variance is distributed like σ^2 with k-2 degrees of freedom. Tolerance levels and confidence intervals are obtained from (39) by assigning the desired probabilities to these levels and intervals. Let us first find the firing (=tolerance) levels P_1 and P_2 such that firing at the P_1-level is "almost certain" while firing at the P_2-level will "almost never" occur. Frequently, these levels are chosen to be $P_1 = 0.999$ and $P_2 = 0.001$. The corresponding tolerance levels V_{P_1} and V_{P_2} are then obtained from (27) or in logarithmic form from (28).

What are the confidence limits for these tolerance levels? We are especially interested in "all-fire" or "no-fire" tests which take one-sided confidence limits. The tolerance levels P_1 and P_2 lie then with a certain probability P_3 below or above stated voltages V_1 and V_2. First, we find the variance for estimates on $\log(P/Q)$:

$$V\{\log(P/Q)\} = \sigma^2 \left[1/k + \log^2(V_P/\bar{V}) \Sigma w / (k \Sigma w \log^2 (V/\bar{V})) \right] \tag{41}$$

where σ^2 is estimated from (39) or (40). This leads to two confidence limits:

$$P \left\{ \log (\Pi/(1-\Pi)) < \log(P_1/(1-P_1)) + t_{P_3} \sigma \left[1/k + \log^2 (V_P/\bar{V}) \Sigma w / (k \Sigma w \log^2(V/\bar{V})) \right]^{1/2} \right\} = P_3 \tag{42}$$

and

$$P \left\{ \log (\Pi/(1-\Pi)) > \log(P_2/(1-P_2)) - t_{P_3} \sigma \left[1/k + \log^2 (V_P/\bar{V}) \Sigma w / (k \Sigma w \log^2(V/\bar{V})) \right]^{1/2} \right\} = P_3 \tag{43}$$

These equations state that with assigned probability P_3 we can find a value of t_{P_3} such that the true value of $\log(\Pi/(1-\Pi))$ will be smaller or larger than the respective upper or lower bounds in the probability statements. These bounds yield the tolerance levels V_{P_1,P_3} and V_{P_2,P_3} :

$$\log V_{P_1, P_3} = \Big[\log (P_1/(1-P_1)) - a_1 + t_{P_3} \sigma \Big\{ 1/k + \log^2 (V_p/\bar{V}) \Sigma w / (k \Sigma w \log^2(V / \bar{V})) \Big\}^{1/2} \Big] / b_1 \tag{44}$$

$$\log V_{P_2, P_3} = \Big[\log(P_2/(1-P_2)) - a_1 - t_{P_3} \sigma \Big\{ 1/k + \log^2(V_p/\bar{V}) \Sigma w / (k \Sigma w \log^2 (V / \bar{V})) \Big\}^{1/2} \Big] / b_1 \tag{45}$$

2. THE NORMAL PROBABILITY (PROBIT) MODEL

The cumulative normal probability function is defined by

$$\phi(u) = (2\Pi)^{-1/2} \int_{-\infty}^{u} \exp(-u^2/2)\, du = P. \tag{46}$$

The inverse cannot be written explicitly and is defined symbolically as

$$u = u(P) \tag{47}$$

As before, we have the model

$$u(\Pi) = \alpha_2 + \beta_2 \log V \tag{48}$$

with estimates

$$u(P) = a_2 + b_2 \log V \tag{49}$$

The normal equations for the parameter estimates are

$$a_2 \ \Sigma w + b_2 \Sigma \ w \log V = \Sigma w \ u(n_x/n) \qquad (50)$$

$$a_2 \ \Sigma w \log V + b_2 \ \Sigma \ w \log^2 V \ = \ \Sigma \ w \ u(n_x/n) \log V \quad (51)$$

The estimated number of firings is again N_x :

$$N_x = n\,\phi(a_2 + b_2 \log V) \qquad (52)$$

Variance computations are similar to those of the logit model and lead to tolerance levels and confidence limits. First we compute the variance of the number of fires for comparison with other models:

$$\mathcal{V}_2(w) \ = \ k \, \Sigma \ w \ (\ n_x - N_x)^2 \ /((k-2) \ \Sigma \ w \ ) \qquad (53)$$

Then we find the variance of the fittings :

$$\mathcal{V}_2 \ = \ k \ \Sigma w \ [\ u(n_x/n) - u(N_x/n) \ ] \ ^2 \ / \ ((k-2) \ \Sigma \ w \ ) \approx \sigma^2 \qquad (54)$$

$$= k \, [\ \Sigma \ \ w \ u^2(n_x/n) - a_2 \ \Sigma \ w \ u(n_x/n) - b_2 \ \Sigma w \ u(n_x/n) \log V] \ / \ ((k-2) \Sigma \ w)$$

This variance is also distributed like σ^2 with $k-2$ degrees of freedom. The tolerance levels V_P are again obtained for some pre-assigned probabilities P_1 or P_2 from (49) and the corresponding confidence limits are similar to those for the Logit Analysis:

$$\log V_{P_1, P_3} \ = [\ u(P_1) - a_2 + t_{P_3} \ \sigma \ \{ \ 1/k$$

$$+ \log^2(V_P/\bar{V}) \ \Sigma \ w \ / \ (k \ \Sigma \ w \log^2 \ (V \ / \ \bar{V})) \ \}^{1/2}] \ / \ b_2 \qquad (55)$$

$$\log V_{P_2,P_3} = [u(P_2) - a_2 - t_{P_3} \sigma \{1/k + \log^2 (V_p/\bar{V}) \Sigma w / (k \Sigma w \log^2(V / \bar{V}))\}^{1/2}]/ b_2 \qquad (56)$$

3. ANALYSIS OF INITIATOR FIRING DATA

In Table B-1 the number of levels is large enough to warrant the use of the above equations. Of interest are only the "mixed" levels, e.g., levels with both fires and failures. "Pure" levels with only fires or only failures do not enter into the weighted equations.

Table B-1

Firing Data for the T18E4 Carbon Bridge Detonator

V	n_x	n_o	n	n_x/n	log V	$n_x n_o/n$	$\log(n_x/n_o)$	$u(n_x/n)$
75.8	0	2	2	0	1.87967	0	-	-
94.9	1	37	38	0.02631	1.97727	0.97368	- 1.56820	- 1/93803
119.0	31	106	137	0.22627	2.07555	23.98540	- 0.53395	- 0.75120
148.6	101	164	265	0.38113	2.17202	62.50566	- 0.21052	- 0.30252
186.0	162	47	209	0.77511	2.26951	36.43062	+ 0.53742	+ 0.75580
233.0	46	6	52	0.88461	2.36736	5.30769	+ 0.88461	+ 1.19836
292.0	5	2	7	0.71428	2.46538	1.42857	+ 0.39794	+ 0.56594
365.0	1	0	1	1.00000	2.56229	0	-	-

Note: Common logarithms were used in all computations.

Table B-2

Summations, Statistics, and Variances Derived from Data of Table B-1

Statistical Element	$w_1 = n_x n_o / n$	$w_2 = n$	$w_3 = 1$
Σw	130.63162	708	6
$\Sigma w \log V$	286.23851	1549.75988	13.32709
$\Sigma w \log^2 V$	628.08641	3399.39718	29.76834
$\Sigma w \log (n_x / n_o)$	- 2.65036	- 27.42447	- 0.49270
$\Sigma w \log (n_x / n_o) \log V$	- 1.23112	- 20.15123	- 0.37130
$\Sigma w\, u(n_x / n)$	- 4.11080	- 32.48884	- 0.47165
$\Sigma w\, u(n_x / n) \log V$	- 2.65910	- 17.56141	- 0.10074
$\Sigma w \log^2 (n_x / n_o)$	26.90457	246.41879	4.01838
$\Sigma w\, u^2(n_x/n)$	51.80262	440.59327	6.73937
a_1	- 11.36546	- 12.35613	- 9.73082
b_1	5.17763	5.62714	4.34395
a_2	- 15.76991	- 16.58004	-12.71404
b_2	7.18261	7.55681	5.68861
U_1 (w)	143.8	93.7	254.7
U_2 (w)	158.0	124.9	225.7
U_1 (w_3)	49.96	38.26	254.7
U_2 (w_3)	55.43	48.31	225.7
U_1	0.03624	0.04439	0.20923
U_2	0.06976	0.07338	0.32897
$V_{1,P=0.5}$	156.7	157.0	173.8
$V_{2,P=0.5}$	156.9	156.3	171.8

Table B-2 shows the primary statistics derived from the data of Table B-1 and the variances for a comparison of models and fittings. Table B-3 shows the tolerance levels and extreme confidence limits for various firing levels computed from the two models for the same data.

Table B-3

Tolerance Levels and Confidence Limits (in Volts) for Two Models

Firing Levels %	Confidence Level %	Logit Model w_1	w_2	w_3	Probit Normal Model w_1	w_2	w_3
99.9	95	893.8	736.0	1431.1	573.6	507.5	1034.1
99.9	50	594.9	535.6	852.3	422.4	400.8	600.1
99	50	380.7	355.2	500.6	330.7	317.6	440.5
95	50	276.7	264.9	342.3	265.8	258.1	334.3
50	50	156.7	157.0	173.8	156.9	156.3	171.8
5	50	88.7	93.0	88.3	92.6	94.7	88.3
1	50	64.5	69.4	60.3	74.4	76.9	67.0
0.1	50	41.3	46.0	35.4	58.3	61.0	49.2
0.1	95	27.6	33.7	18.0	43.2	48.4	29.3

Note: $u(0.999) = 3.09024$, $u(0.99) = 2.32636$, $u(0.95) = t_{P_3} = 1.64486$

Let us now investigate the validity of the two models. We turn to Table (B-2) and note that only the equally weighted variances $U_1(w_3)$ and $U_2\ (w_3)$ are truly comparable while variances obtained under different weighting procedures may not be used to form variance ratios for F-tests. The smallest variance in the rows $V\ (w_3)$ is 38.26 from the logit model with weights n . This variance has (4,4) degrees of freedom and the other variances are not significantly out of line. Even the weighting procedure w_3 can only be rejected on the basis of professional statistical knowledge and will not be used in future work. Therefore, the only two weighting procedures of interest are w_1 and w_2 and the two models differ little in their description of the quantal response phenomenon, certainly not enough to warrant rejection of either.

Table (B-2) shows also that the estimates of the mean firing levels agree well for both models and the two weighting procedures w_1 and w_2. Unfortunately, the evaluation of detonators is not based on their 50% firing level but on the two extreme levels, one at 99.9% and one at 0.1% firing. The former is the level at which the detonators will almost certainly fire. The certainty is increased if the firing voltage is raised further. The lower, safety level indicates the voltage at which the detonators will almost never fire, and hence are considered safe to handle.

Table (B-3) lists various firing levels obtained from the two models and we shall consider only the weighting procedures w_1 and w_2. The probit model yields a substantially narrower spread than the logit model for the interval from 0.1% to 99.9%. This is not surprising since any estimates obtained from a statistical study depend primarily upon the model used. No quantitative estimates can even be made without a model and its parameters, hence the choice of the model determines the kind of estimates obtained. In other words, if we had more insight into the actual behavior of the detonators, we might know that the probit model, for example, is truly representative and that the upper firing level with 95% confidence lies even below 507.5 volts. On the other hand, if the logit model were truly representative of the actual functioning of the detonators, the upper 99.9% firing level would lie below 736.0 volts with 95% confidence.

Similar extimates may be carried out for the lower, safety level. Thus estimates of extreme firing levels can be made with confidence only if the actual response pattern of the detonators is known. Such information, however, cannot be gleaned from Bruceton or probit type experiments. The only experimental technique known which gives a maximum amount of information about these extreme levels is that developed by Bartlett. This experimental design requires an enormous number of units and is very costly; however, it cannot be avoided if we are to settle the model problem. It seems imperative that in a few cases, at least, the response of detonators at extreme firing levels must be determined by an actual firing test so that a decision can be reached as to which type model is representative of the "true" detonator response.

4. THEORETICAL CONSIDERATIONS CONCERNING LOGIT AND PROBIT MODELS

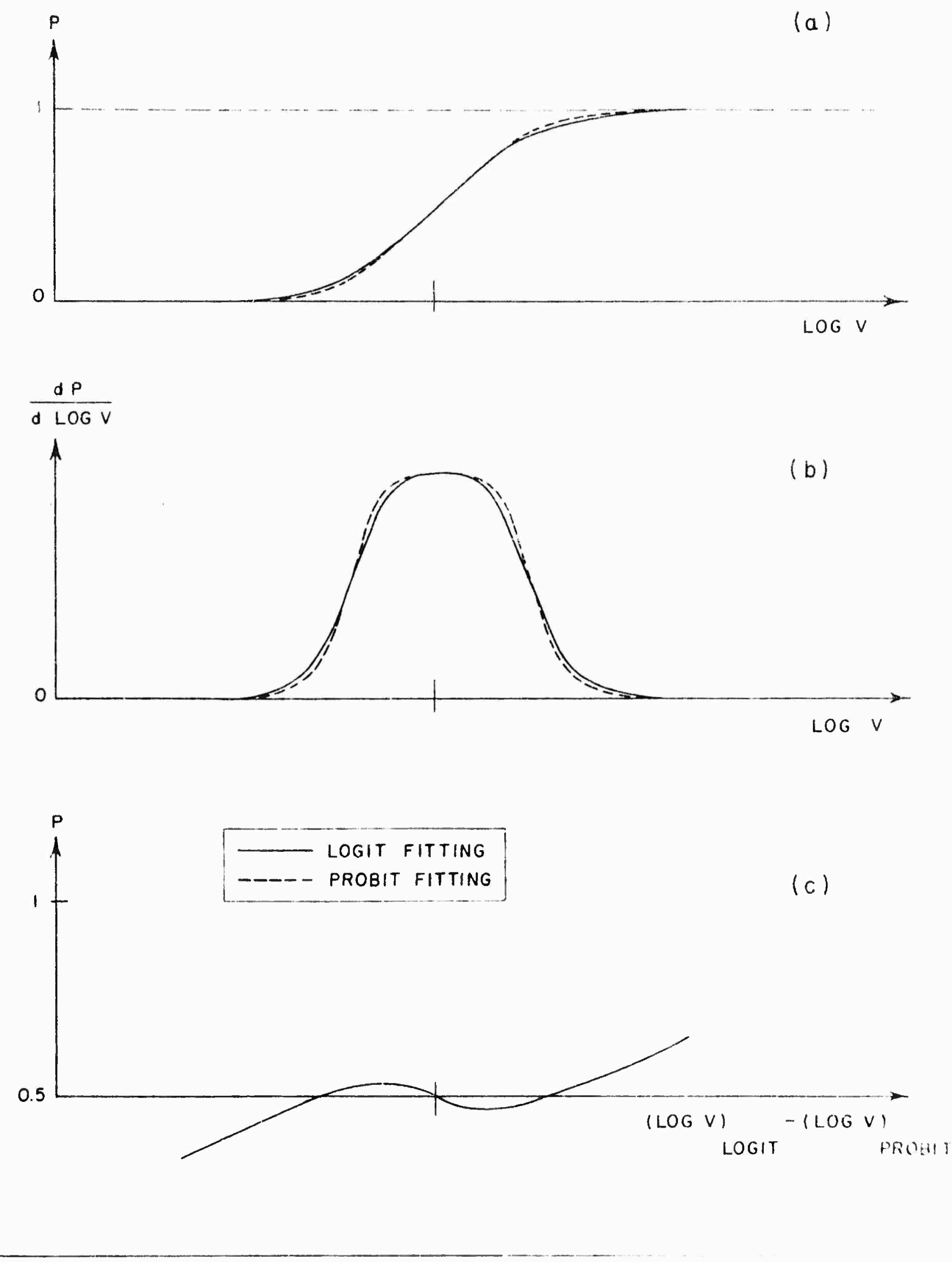
Neither model considered here is really based on any physical theory. The detonator response curves, like so many other quantal response phenomena, just look like sigmoids, and the natural procedure was to use simple sigmoid models in their evaluation. A more penetrating analysis of these models can be made, however.

Figure (B-2a) shows once more a typical sigmoid quantal response curve, fitted to a fictitious set of data. It is assumed that the fitting was carried out by logit and probit methods. The two fittings do not differ very much from one another, certainly less than the experimental data differ from either fitting. Therefore, the "true" model cannot be ascertained, unless observations at the extreme firing levels are available.

The two fittings represent smooth curves and their derivatives are shown in Figure (B-2b). The two derivative curves again differ little; the data are not sufficient to distinguish between the curves with high probability. Estimates may now be made from these two models for extreme firing levels after the fittings are made. Since both models contain two parameters, the estimates can be made to agree in the location of the mean firing level and in the slope of the sigmoid response curve at this mean firing level. However, agreement may be forced with respect to any other two desired properties of the two models.

Estimates of the firing levels that are far from the mean will be more in error than those near the mean. Figure (B-2c) shows these differences qualitatively. If one of the models is "correct" estimates

DIFFERENCE IN ESTIMATES FROM LOGIT AND PROBIT MODELS



REPORT I-1804-1 FIGURE [illegible]

made from the other model will be increasingly incorrect as higher or lower firing levels are chosen.

The mathematical analysis proceeds from equation (29) which shows that the slope of the logit sigmoid is:

$$\frac{d\ \Pi}{d \log V} = \beta_1 \exp(\alpha_1 + \beta_1 \log V) / [1 + \exp(\alpha_1 + \beta_1 \log V)]^2 \qquad (57)$$

while the slope of the probit sigmoid obtained from (46) is:

$$\frac{d\ \Pi}{d \log V} = \frac{\beta_2}{\sqrt{2\Pi}} \exp\left(-(\alpha_2 + \beta_2 \log V)^2 / 2\right) \qquad (58)$$

If the two sigmoids are to agree in the mean,

$$\alpha_1 / \beta_1 = \alpha_2 / \beta_2 \qquad (59)$$

The slope at the mean of the logit sigmoid is

$$\left(\frac{d\ \Pi}{d \log V}\right)_{P = 1/2} = \beta_1 / 4 \qquad (60)$$

while the slope at the mean of the probit sigmoid is

$$\left(\frac{d\ \Pi}{d \log V}\right)_{P = 1/2} = \beta_2 / \sqrt{2\Pi} \qquad (61)$$

Combination of conditions (59) and (62)

$$\beta_1 / 4 = \beta_2 / \sqrt{2\Pi} \qquad (62)$$

yields for a given set of (α_1, β_1)

$$\alpha_2 = \alpha_1 \sqrt{2\Pi} / 4 \tag{63}$$

$$\beta_2 = \beta_1 \sqrt{2\Pi} / 4 \tag{64}$$

and the difference in estimates for $\log V$ becomes

$$(\log V)_{\text{Logit}} - (\log V)_{\text{Probit}} = \frac{-\alpha_1 + \log(\Pi/(1-\Pi))}{\beta_1} + \frac{\alpha_2 - u(\Pi)}{\beta_2}$$

$$= \frac{1}{\beta_1} [\log (\Pi/(1-\Pi)) - 4\, u(\Pi) / \sqrt{2\Pi}\,] \tag{65}$$

$$= \frac{1}{\beta_1} [\log \frac{\Pi}{1-\Pi} - \frac{\log \frac{P_x}{1-P_x}}{u(P_x)} u(\Pi)\,] \tag{66}$$

where $P_x = 0.5$. This last transformation shows that agreement of logit and probit models at the mean, and at the slope of the mean, is merely a special case of a more general class of conditions which can be imposed upon the models.

The models can be made to agree in their means and in the cumulants at (P) and (1-P). This procedure yields a matched range such that the two models agree in three points, as shown by the differences of Figure (B-2c). The conditions to be satisfied are (59) and for some fixed P_x :

$$\frac{\alpha_1 + \log P_x / Q_x}{\beta_1} = \frac{\alpha_2 + u(P_x)}{\beta_2} \tag{67}$$

which yields

$$\alpha_2 = \alpha_1 \, u(P_x) \,/\, \log(P_x/Q_x) \tag{68}$$

$$\beta_2 = \beta_1 \, u(P_x) \,/\, \log(P_x/Q_x) \tag{69}$$

and agrees in form exactly with (66) if we consider the difference between the estimates for $\log V$:

$$(\log V)_{\text{Logit}} - (\text{Log } V)_{\text{Probit}} = \frac{1}{\beta_1}\left[\log \frac{\Pi}{1-\Pi} - \frac{\log \frac{P_x}{1-P_x}}{u(P_x)} \, u(\Pi)\right] \tag{70}$$

Since the condition that the two models agree in the mean and the slope at the mean is only a special case of the more general condition that the two models agree in the mean and in two selected cumulant points, we have tabulated a few values for probabilities ranging from extremely low values to extremely high values. Table (B-4) shows that the matched range can be varied at will by assigning to P_x suitable values. Inside that range, logit estimates are smaller than probit estimates if the probabilities exceed one-half; the logit estimates are larger than the probit estimates if the probabilities are smaller than one-half. The picture is reversed outside the matching range and the differences between probit and logit estimates tend to infinity as P approaches unity; they tend to negative infinity as P approaches zero.

Table (B-4)

Difference Between Estimates of log V for Selected Probability Values

Π	β_1 [(log V) $_{Logit}$ − (log V) $_{Probit}$]		
	$P_x = 0.5$	$P_x = 0.9$	$P_x = 0.99$
0.9999	1.4226	1.2308	0.8096
0.999	0.8579	0.6986	0.3486
0.99	0.3834	0.2634	0
0.9	0.0661	0	-0.1451
0.8	0.0189	-0.0320	-0.1285
0.5	0	0	0
0.2	-0.0189	0.0320	0.1285
0.1	-0.0661	0	0.1451
0.01	-0.3834	-0.2634	0
0.001	-0.8579	-0.6986	-0.3486
0.0001	-1.4226	-1.2308	-0.8096

Note: For the sake of convenience, logarithms used here are common logarithms.

5. SUMMARY AND CONCLUSIONS

The preceding study summarizes for reference purposes many of the equations necessary to carry out precise analyses of quantal response data for the logit and the probit models. Estimates for extremely high or low firing levels from the two models differ considerably, while estimates of the mean level differ only slightly. This is in part caused by the use of Bruceton type data which concentrate all information near the mean firing level. Fully as much of the uncertainty in extreme level estimates is caused by the characteristic variations of the two models themselves. The total spread between extreme level estimates is considerably larger for the logit analysis than for the probit analysis. This fact should be borne in mind when comparisons of extreme level firing voltages are to be made.

Summarizing the results of this study, we come to the following conclusions:

(a) Bruceton type experimental designs are not sufficiently strong to discriminate between probit and logit models.

(b) Estimates of extreme firing levels made from the two models differ considerably in magnitude; the probit model yields the narrower tolerance bands.

Selected References

1. A. H. Bowker, Tolerance Limits for Normal Distributions, Chapter 2, Techniques of Statistical Analysis, New York, 1947.
2. W. Kaplan, Advanced Calculus, Addison Wesley Press, Inc., 1952.
3. A summary of Known Distribution Functions, Verteilungsfunktionem und ihre Auszeichnung durch Funktionalgleichungen, Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker, vol. 45 (1945), pp. 97-163, by B. Haller, Translated by R. E. Kalaba, T-27, 7 January 1953.

APPENDIX C

COMPUT/ ⁻ONAL PROCEDURE FOR SENSITIVITY ANALYSIS

APPENDIX C

COMPUTATIONAL PROCEDURE FOR SENSITIVITY ANALYSIS

In this appendix we shall describe the computational procedure used to derive the parameters which appear in the mathematical models for detonator sensitivity analysis. The models, described in Section 4.6, can be written in logarithmic form:

$$F(P) = a + b \log V + c_1 \log C + c_2 \log^2 C + c_3 \log^3 C + \dots \tag{71}$$

According to Appendix B, the probability function F(P) may be of the logit or the probit type. The number of parameters chosen in the fitting process does not affect the procedure as such, and we shall list here the equations that arise for fittings with the five parameters stated explicitly above. Since quantal response data must be weighted (cf. Appendix B) the least squares equations require weighting factors of the form $w = n_o n_x / n$ where n_o is the number of observed misfires, n_x is the number of observed fires, and $n = n_o + n_x$ is the total number of detonators tested for a specific combination of firing voltage and firing capacitance. For the logit model, the least-squares equations are, then, written in matrix form:

$$\begin{bmatrix} \Sigma w & \Sigma w \log V & \Sigma w \log C & \Sigma w \log^2 C & \Sigma w \log^3 C \\ \Sigma w \log V & \Sigma w \log^2 V & \Sigma w \log C \log V & \Sigma w \log^2 C \log V & \Sigma w \log^3 C \log V \\ \Sigma w \log C & \Sigma w \log V \log C & \Sigma w \log^2 C & \Sigma w \log^3 C & \Sigma w \log^4 C \\ \Sigma w \log^2 C & \Sigma w \log V \log^2 C & \Sigma w \log^3 C & \Sigma w \log^4 C & \Sigma w \log^5 C \\ \Sigma w \log^3 C & \Sigma w \log V \log^3 C & \Sigma w \log^4 C & \Sigma w \log^5 C & \Sigma w \log^6 C \end{bmatrix} \begin{bmatrix} a \\ b \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$= \begin{bmatrix} \Sigma w \log (n_x/n_o) \\ \Sigma w \log (n_x/n_o) \log V \\ \Sigma w \log (n_x/n_o) \log C \\ \Sigma w \log (n_x/n_o) \log^2 C \\ \Sigma w \log (n_x/n_o) \log^3 C \end{bmatrix} \tag{72}$$

For the probit type analysis, the right-hand side-column vector is replaced by:

$$\begin{bmatrix} \Sigma w\ u\ \ (n_x/n) & \\ \Sigma w\ u\ \ (n_y/n) & \log V \\ \Sigma w\ u\ \ (n_x/n) & \log C \\ \Sigma w\ u\ \ (n_x/n) & \log^2 C \\ \Sigma w\ u\ \ (n_x/n) & \log^3 C \end{bmatrix} \tag{73}$$

The first two of these equations are similar to equations (34), (35), (50), and (51) shown in Appendix B.

The variance for this fitting $\upsilon\{p\}$ is obtained from

$$\begin{aligned}(N-6)\ \upsilon\{p\} &= \Sigma w \log^2 (n_x/n_o) - a\ \Sigma w \log (n_x/n_o) \\ &- b\ \Sigma w \log (n_x/n_o) \log V - C_1\ \Sigma w \log (n_x/n_o) \log C \\ &- c_2\ \Sigma w \log (n_x/n_o) \log^2 C - C_3\ \Sigma w \log (n_x/n_o) \log^3 C\end{aligned} \tag{74}$$

Table (C-1) contains the basic data that were used to carry out these computations. In addition, the table holds the summations that were needed to solve equations (72) and (73). The solution led to the following values for the five parameters:

Parameter	Logit Equations	Probit Equations
a	− 6.03338	− 8.39311
b	+ 6.34594	+ 8.84339
c_1	+ 1.03687	+ 1.46126
c_2	− 0.392313	− 0.536254
c_3	− 0.00485608	− 0.00476236
υ	0.00641	0.0122

Table (C-1) ORIGINAL DATA AND SUMMATIONS USED IN 5-POINT FITTING OF LOGIT AND PROBIT MODELS FOR T18E4 (R < 1000 Ω : SPECIAL) CARBON BRIDGE DETONATOR QUANTAL RESPONSE DATA

n_o	n_x	log V	log C	$\frac{n_o\,n_x}{(n_o + n_x)} = w$	log n_x/n_o	$u(n_x/n)$
7	4	2.4750	-4.00000	2.54545	-.24304	- .34875
9	6	2.5500	-4. -	3.60000	-.17609	- .25335
1	8	2.6250	-4. -	.88889	.90309	+1.22064
7	1	1.87500	-3.0000	.87500	-.84512	-1.15035
6	7	1.9500	-3. -	3.23076	.06696	+ .096556
4	6	2.0250	-3. -	2.40000	.17609	+ .25345
2	4	2.1000	-3. -	1.33333	.30103	+ .43073
9	8	1.5000	-2.0000	4.23529	-.05115	- .073854
3	9	1.5750	-2. -	2.25000	.47712	+ .67449
11	3	1.1250	-1.0000	2.35714	-.56428	- .79164
6	10	1.2000	-1.	3.75000	.21186	+ .31864
13	2	.9000	0	1.73333	.81293	-1.11077
4	12	.9750	0	3.00000	.47712	+ .67449
1	4	1.0500	0	.80000	.60206	+ .84168

N = 14

$\Sigma\ w = 32.99919$

$\Sigma\ w \log V = 55.7873895$

$\Sigma\ w \log C = -70.73235$

$\Sigma\ w \log^2 C = +\ 215.14955$

$\Sigma\ w \log^3 C = -\ 719.84265$

$\Sigma\ w \log^4 C = +2545.62911$

$\Sigma\ w \log^5 C = -9321.69945$

$\Sigma\ w \log^6 C = +34,948.51895$

$\Sigma\ w \log^2 V = 104.841038250$

$\Sigma\ w \log V \log^2 C = 472.15712250$

$\Sigma\ w \log V \log C = -145.00025650$

$\Sigma\ w \log V \log^3 C = -1047.59426250$

$\Sigma\ w \log n_x/n_o = +\ .676222522$

$\Sigma\ w \log n_x/n_o \log V = 1.149559840275$

$\Sigma\ w \log n_x/n_o \log C = -.28387941$

$\Sigma\ w \log n_x/n_o \log^2 C = -1.597672083$

$\Sigma\ w \log n_x/n_o \log^3 C = 14.346491963$

$\Sigma\ w \log^2 n_x/n_o = 5.3821445921$

$\Sigma\ w \log^4 C \log n_x/n_o = -77.6119711779$

$\Sigma\ w \log^4 C \log V = +5989.358542500$

$\Sigma\ w \log^7 C = -133,230.94065$

$\Sigma\ w \log^8 C = +514,101.11711$

$\Sigma\ wu = 1.078410305$

$\Sigma\ wu \log V = 1.6868847333000$

$\Sigma\ wu \log C = -.343397096$

$\Sigma\ wu \log^2 C = -2.896332750$

$\Sigma\ wu \log^3 C = +23.602426930$

$\Sigma\ wu^2 = 10.42943466157164$

These parameters were used to construct Figures (4-8) and (4-9) by substituting the desired probability values in equation (71).

To avoid the possibility of a misunderstanding, it should be pointed out that the two variances reported above are obtained for the respective models. Hence, they cannot be tested against each other, but only against similar variances obtained from the same data for models with either more or fewer parameters. Since no other fittings were computed, we are not in a position to state positively whether this is the most significant fit to the data. However, the fact that c_3 almost vanishes in both fittings, leads us to believe that the introduction of an additional parameter c_4 would not reduce these variances significantly.

APPENDIX D

OPERATING CHARACTERISTICS FOR VARIOUS SAMPLING PLANS

APPENDIX D

OPERATING CHARACTERISTICS FOR VARIOUS SAMPLING PLANS

Acceptance sampling plans and their operating characteristics have been under study for some time. A number of treatises explain their probability basis in detail. Therefore, we shall cite here formulas only for the various plans outlined in Section 5.2. A distinction is always made between sampling plans from a finite population that are made either with or without replacement. The former leads to the simple, binomial formulas, while the latter requires the substantially more correct hypergeometric distribution. For small samples from large lots, as in our case, the binomial yields practically the same results as the hypergeometric. For the requisite error formulas, see reference (18). We shall use the following notation:

P_a = Probability that a lot be accepted on the basis of some acceptance sampling plan,

p = Probability that a sample unit fails at the chosen test level (fraction defective),

$q = 1 - p$ = Probability that a sample unit will not fail at the chosen test level,

p_o = Average fraction defective permissible; the operating level of the sampling plan,

n_i = Number of units tested during the i-th stage of a sampling plan with k stages $(i=1,2,\ldots,k)$,

f_{ai} = Cumulative largest number of failures permitting acceptance of lot at the end of the i-th stage,

f_{ri} = Cumulative smallest number of failures forcing rejection of the lot at the end of the i-th stage,

$f_{ak} = f_{rk} - 1$ = (necessary) terminal condition of a sampling plan with k stages,

N_a = Average number of units tested in the long run under some sampling plan, leading to lot acceptance,

N_r = Average number of units tested in the long run under some sampling plan, leading to lot rejection,

$N = N_a + N_r$ = Average number of units tested in the long run under some sampling plan.

In Section 5.2, three sampling plans from reference (23) were compared with the present sampling plan. The evaluation of these four plans is based upon the following formulas:

1. SINGLE SAMPLING, NORMAL INSPECTION, $p_o = 0.001$

This plan has one stage, $(k = 1)$ with $n_1 = 300$, $f_{a1} = f_{r1} - 1 = 1$. For average outgoing quality with a fraction p defective, we find:

$$P_a = q^{300} + 300\ p\ q^{299} \tag{75}$$

$$N_a = 300\ q^{300} + 90{,}000\ p\ q^{299} \tag{76}$$

$$N_r = 300 - 300\ q^{300} - 90{,}000\ p\ q^{299} \tag{77}$$

$$N = 300 \tag{78}$$

These formulas were used to compute the curve shown in Figure (5-2) and the entry on page 91.

2. DOUBLE SAMPLING, NORMAL INSPECTION, $p_o = 0.001$

This plan has $k = 2$ stages. The individual stages require $n_1 = 200$ and $n_2 = 400$ units. The final decision is made at the end of the second stage where $f_{a2} = f_{r2} - 1 = 2$. Rejection during the first stage occurs for $f_{r1} = 3$. But if only two of the first two-hundred units misfire, acceptance is only permissible if no additional units misfire during the testing of the next four-hundred units. We find:

$$P_a = q^{200} + 200\ p\ q^{599} + 99{,}900\ p^2\ q^{598} \tag{79}$$

$$N_a = 200\ q^{200} + 120{,}000\ p\ q^{599} + 59{,}940{,}000\ p^2\ q^{598} \tag{80}$$

$$N_r = 200 - 200\ q^{200} + 80{,}000\ p\ q^{199} + 7{,}960{,}000\ p^2\ q^{198} - 120{,}000\ p\ q^{599} - 59{,}940{,}000\ p^2\ q^{598} \tag{81}$$

$$N = 200 + 80{,}000\ p\ q^{199} + 7{,}960{,}000\ p^2\ q^{198} \tag{82}$$

These formulas were used to compute the curve of Figure 5-2 and the entry for N on page 91.

3. MULTIPLE SAMPLING, NORMAL INSPECTION, $p_o = 0.001$

This plan has $k = 7$ stages. Each stage requires $n_i = 75$ units $(i = 1,2,\ldots,7)$. Rejection may occur during the first stage if $f_{r1} = 2$; acceptance is only permissible after the third stage if $f_{a3}=0$. The final decision at the seventh stage is based upon $f_{a7} = f_{r7} - 1=3$. The binomial leads to

$$P_a = q^{225} + 225\ p\ q^{374} + 33{,}750\ p^2\ q^{448}\ , \tag{83}$$

$$N_a = 225\ q^{225} + 84{,}375\ p\ q^{374} + 15{,}187{,}500\ p^2\ q^{448} + 2{,}321{,}156{,}250\ p^3\ q^{522}\ , \tag{84}$$

$$\begin{aligned}N_r = 75 &- 5625\ p\ q^{74} + 75\ q^{75} + 11{,}250\ p\ q^{149} + 75\ q^{150} + 16{,}875\ p\ q^{224} \\ &- 225\ q^{225} + 1{,}265{,}625\ p^2\ q^{298} + 16{,}875\ p\ q^{299} + 141{,}750{,}000\ p^3\ q^{372} \\ &+ 2{,}531{,}250\ p^2\ q^{373} - 84{,}375\ p\ q^{374} + 331{,}593{,}750\ p^3\ q^{447} \\ &- 15{,}187500\ p^2\ q^{448} - 2{,}321{,}156{,}250\ p^3\ q^{522}\end{aligned} \tag{85}$$

$$N = 75 - 5625\,p\,q^{74} + 75\,q^{75} + 11{,}250\,p\,q^{149} + 75\,q^{150} + 16{,}875\,p\,q^{224}$$
$$+ 1{,}265{,}625\,p^2\,q^{298} + 16{,}875\,p\,q^{299} + 141{,}750{,}000\,p^3\,q^{372}$$
$$+ 2{,}531{,}250\,p^2\,q^{373} + 331{,}593{,}750\,p^3\,q^{447} \tag{86}$$

These formulas were used to compute the curve of Figure 5-2 and the entry for N on page 91.

4. PRESENT ACCEPTANCE SAMPLING PLAN

The present acceptance sampling plan is essentially a double sampling plan with $n_1 = 50$, $n_2 = 100$. Acceptance is permissible if $f_{a1} = 1$, $f_{a2} = 2$; rejection is required if $f_{r1} = 3$, $f_{r2} = 3$. We find:

$$P_a = q^{50} + 50\,p\,q^{49} + 1225\,p^2\,q^{148}\,, \tag{87}$$

$$N_a = 50\,q^{50} + 2500\,p\,q^{49} + 183{,}750\,p^2\,q^{148} \tag{88}$$

$$N_r = 50 - 50\,q^{50} + 122{,}500\,p^2\,q^{48} - 2500\,p\,q^{49} - 183{,}750\,p^2\,q^{148} \tag{89}$$

$$N = 50 + 122{,}500\,p^2\,q^{48} \tag{90}$$

These formulas were used to compute the curve of figure 5-2 and the entry on page 91.

5. MODIFICATION OF SEQUENTIAL SAMPLING PLANS

The figures computed for the several sampling plans permit a minor modification. It is tacitly assumed that all samples are always tested completely, even if the number of failures should be excessive. However, a lot might be rejected before a sample has been tested completely. For example, the single sampling plan would permit rejection after two failures have occurred among the first 200 units. The above computed average of $N = 300$ would thus become a trifle smaller. Occasionally, rejection will take place prior to the testing of 300 units. The probability of such an occurrance is very small and the resultant change in the value of N is minute. But this fact pertains to sampling plans of this nature and warrants inclusion at this point. A general analysis of sampling plans falls outside the scope of this report.

APPENDIX E

TABLE E-1 FUNCTIONING TIME DATA FOR THE T18E3 (AAP-50-2) CARBON BRIDGE DETONATORS

Table E-1. FUNCTIONING TIMES OBSERVED WITH THE T18E3 (AAP-50-2) CARBON BRIDGE DETONATOR

C = Firing Capacity, µf; V = Firing Voltage t = Functioning Time, µsec.

C	V	t
.000217	200	3.125
	252	2.750
		3.000
		3.000
		3.000
		3.000
		3.125
		3.125
		3.250
	317	3.125
		3.250
		3.250
		3.250
.0005	126	2.750
		3.000
	159	2.875
		2.875
		3.000
		3.125
		3.125
		3.125
		3.125
		3.375
	200	2.375
		2.625
		2.625
		2.750
		2.875
		3.000
		3.125
		3.125
	240	3.250
		2.500
		2.375
		2.500
		2.875
	252	2.250
		2.750
		2.750
		2.875
.0005	252	3.000
	480	2.750
		2.125
		2.250
		2.500
		2.375
	960	3.375
		2.125
		2.500
		2.375
		2.125
.001	100	3.375
	126	2.375
		2.625
		2.625
		2.625
		3.000
		3.000
		3.125
		3.250
	159	2.625
		2.625
		2.875
		3.000
		3.000
		3.250
		3.375
		3.625
	160	3.500
		2.750
		3.250
		2.375
		2.750
	200	3.250
	320	3.250
		2.875
		2.875
		2.875
		2.875
	640	3.125
		3.000
.001	640	2.625
		2.375
		2.875
.00197	100	3.375
		3.500
		3.625
		3.625
	126	3.000
		3.500
		3.625
.005	63.2	3.375
		3.375
		3.625
		4.125
		5.375
	79.5	3.125
		3.250
		3.375
		3.375
		3.500
		3.875
		7.375
		11.250
	100	3.000
		3.125
		3.125
	120	2.500
		2.750
		2.625
		2.250
		2.250
	240	2.625
		2.375
		2.375
		2.500
		10.000
	480	3.125
		2.625
		2.250
		2.250

Table E-1. FUNCTIONING TIMES OBSERVED WITH THE T18E3 (AAP-50-2) CARBON BRIDGE DETONATOR (Cont.)

C = Firing Capacity, µf; V = Firing Voltage, t = Functioning Time, µsec.

C	V	t
.005	480	2.375
	960	2.625
		3.000
		2.625
		2.750
.01	39.9	6.875
		9.750
	50.2	4.500
		5.250
		14.500
	63.2	2.875
		3.500
	79.5	2.625
		2.875
		3.125
	79.5	3.375
		3.875
	100	3.375
	160	3.375
		3.375
		2.125
		2.875
	320	3.125
		2.500
		2.375
		2.625
		2.750
	640	3.125
		3.000
		3.000
		2.625
		2.750
.0196	31.7	11.000
	39.9	3.125
		4.750
		5.000
		5.250
		5.250
		6.000
		17.500
.0196	39.9	18.375
		22.625
	50.2	4.250
		4.875
		4.875
		6.250
		8.625
		9.500
		15.875
		26.000
	63.2	3.250
		30.625
		30.875
.05	25.2	3.750
		4.625
		5.000
		5.750
		8.625
		17.250
		18.125
	31.7	5.875
		9.000
		12.875
		17.875
		19.000
		32.250
		43.875
	39.9	7.000
		8.875
		9.125
		29.125
		34.875
		37.375
	120	3.500
		2.875
		3.250
		2.875
		2.500
	240	2.750
		2.500
.05	240	2.875
		2.750
		2.625
	480	2.750
		2.875
		3.250
		2.375
.1	20	24.375
	25.2	10.125
		25.625
		46.250
		54.375
		86.875
	31.7	15.875
	40	6.375
		7.000
		12.250
		6.750
		55.875
	60	3.625
		2.750
		3.875
	80	3.125
		3.375
		2.750
		2.500
		2.750
	160	3.375
		3.125
		2.375
		3.375
		2.250
	320	3.000
		2.750
		2.875
		2.625
		2.750
.215	25.2	9.375
		22.875
		27.250

Table E-1. FUNCTIONING TIMES OBSERVED WITH THE T18E3 (AAP-50-2) CARBON BRIDGE DETONATOR (Cont.)

C = Firing Capacity, µf; V = Firing Voltage; t = Functioning Time, µsec.

C	V	t	C	V	t
.215	25.2	28.375	.5	480	2.375
		129.625			
			1	12.6	18.875
.5	12.6	9.000			113.250
		33.750		20	35.500
	15.9	33.750			59.500
		38.875			88.750
	20	35.625			113.750
		63.000			169.875
	25.2	7.000			291.250
		15.625		40	10.000
	40	2.500			7.750
		3.125			19.625
		3.750			7.000
		20.625			22.125
	60	2.750		60	3.375
		2.750			2.875
		3.000			6.625
		17.625			6.500
		2.250		80	2.750
	80	2.500			7.500
		2.375			2.625
		2.750			2.500
		5.375			2.500
	100	2.250		100	2.625
		2.500			
		3.125	1	100	2.250
		3.250			2.750
	120	3.500			2.625
		2.750		160	3.125
		2.500			2.875
		2.875			2.625
		2.500			2.875
	240	2.875			2.625
		2.625		320	2.875
		2.250			3.000
		2.750			3.000
		2.375			2.500
	480	3.625			2.625
		2.875			
		2.625			
		2.500			

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